Reputational Economies of Scale

Daniel Klerman*

Abstract

For many years, most people working in law and economics have assumed that the strength of reputational constraints is positively correlated with the frequency of repeat play. Firms that sell more products or services are more likely to be trustworthy than those that sell less, because they have more to lose if consumers decide they have behaved badly. That assumption has been called into question by recent work by Rasmusen and Iacobucci, who argue that, under the standard infinitely repeated game model of reputation, reputational economies of scale will occur only under special conditions, such as monopoly, because larger firms not only have more to lose from behaving badly, but also more to gain. This article argues that reputational economies of scale exist under the infinitely repeated game model of reputation even when there is competition and without other special conditions, as long as low quality is detected with probability less than one and as long as the probability of detection is positively correlated with the frequency of repeat play. With similar assumptions, a finite horizon model of reputation also produces reputational economies of scale.

I. Introduction

For many years, most people working in law and economics have assumed that the strength of reputational constraints is positively correlated with the frequency of repeat play. Firms that sell more products or services are more likely to be trustworthy than those that sell less, because they have more to lose if consumers decide they have behaved badly. This assumption helps explain why law and accounting firms can act as gatekeepers,[1] why mass

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* Edward G. Lewis Professor of Law & History, USC Law School. dklerman@law.usc.edu. The author thanks Yeheskel Bar-Isaac, Ed Iacobucci, Alex Lee, Bentley MacLeod, Eric Rasmusen, Steven Shavell, Steven Tadelis and participants at the American Law & Economics Association 2012 annual meeting for their comments and suggestions. The author also thanks Timothy Hughes and Jeffrey Zhao for their help on Section V and Appendices A and B.

[1] Reinier H. Kraakman, Corporate Liability Strategies and the Costs of Legal Controls, 93 Yale L.J. 857, 891 (1984); John C. Coffee, Jr., Gatekeepers: The Professions and Corporate Governance 138 (2006); Stephen Choi, Market Lessons for Gatekeepers, 92 Nw. U. L. Rev. 916, 918 (1998); Jonathan Barnett, "Intermediaries Revisited: Is Efficient Certification Consistent with Profit Maximization?" 37 J. Corp. Law 475 (2012). For a skeptical view of lawyers and accountants as gatekeepers, see Johnathan Macey, *The Death of Corporate Reputation* (2013), chapters 6 and 7.

market products are more likely to be safe,[2] why firms are less likely to exploit one-sided contracts than consumers,[3] and why manufacturers market new products under the umbrella of established trademarks.

Nevertheless, recent articles by Eric Rasmusen and Edward Iacobucci call into question the assumption of reputational economies of scale.[4] They assert that, under the infinitely repeated game model of reputational enforcement, there is no advantage to firms that sell products with greater frequency. While firms that sell more have more to lose if they misbehave, they also have more to gain from misbehaving, and these two effects offset each other precisely. Instead, Rasmusen and Iacobucci assert that there are reputational economies of scale only under special circumstances, such as monopoly. Rasmusen and Iacobucci's work is consistent with prior work that shows reputational economies, because those articles assume an infinite horizon model in which sellers are able to price monopolistically.[5]

This article argues that reputational economies of scale exist under the infinitely repeated game model of reputation, even when there is competition and without other special conditions. In addition, this article shows that there are reputational economies of scale under a finite-horizon model of reputation with two types similar to that developed by Kreps and Wilson.[6]

The infinitely repeated game reputation model requires only minor adjustment in order to generate reputational economics of scale. The only modification necessary is to assume that low

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[2] A. Mitchell Polinsky & Steven Shavell, The Uneasy Case for Products Liability, 123 Harv. L. Rev. 1437, 1491 (2010).

[3] Lucian A. Bebchuk & Richard A. Posner, One-Sided Contracts in Competitive Consumer Markets, 104 Mich. L. Rev. 827, 832 (2006).

[4] Edward Iacobucci, Reputational Economies of Scale, with Application to Law Firms, 14 American Law & Economics Review 302 (2012); Eric Rasmusen, Leveraging of Reputation through Umbrella Branding: The Implications for Market Structure, 26 Journal of Economics and Management 261 (2016).

[5] Rafael Rob & Arthur Fishman, Is Bigger Better? Customer Base Expansion through Word-of-Mouth Reputation," 113 J. Political Econ. 1146-1162 (2005); Hongbin Cai & Ichiro Obara, Firm Reputation and Horizontal Integration, 40 RAND J. Econ. 340-63 (2009); Jay Pil Choi, Brand Extension as Informational Leverage, 65 Rev. Econ. Stud. 655-69 (1998); Birger Wernerfelt, Umbrella Branding as a Signal of Product Quality: An Example of Signaling by Posting a Bond, 19 RAND J. Econ 458-66 (1988). Rogerson (1983) shows that higher quality firms tend to have more customers in a model with competition, although he assumes that firms "make a once-and-for-all quality choice upon entering the market," and therefore do not face that moral hazard problem (temptation to shirk on quality) that characterizes most of the literature. William Rogerson, Reputation and Product Quality, 14 Bell J. Econ 508, 510 (1983). Helpful surveys on reputation include Heski Bar-Isaac & Steven Tadelis, Seller Reputation (2008); W. Bentley MacLeod, Reputations, Relationships and Contract Enforcement, 45 J. Econ. Lit 595-628 (2007); George J. Mailath & Larry Samuelson, Repeated Games and Reputations: Long-Run Relationships (2006).

[6] David Kreps & Robert Wilson, Reputation and Imperfect Information, 27 J. Econ Theory 253-79 (1982). For a similar model with an infinite number of types, see Paul Milgrom & John Roberts, Predation, Reputation and Entry Deterrence, 27 J. Econ Theory 280-312 (1982).

quality is detected with probability less than one and that the probability of detection is positively correlated with the frequency of repeat play. This assumption is valid for nearly all situations to which reputational enforcement is usually applied. For example, if a manufacturer skimps on the safety of its products, the probability that any one product will cause an accident is likely to be less than one. Nevertheless, if it sells many products, it is quite likely that at least one product will cause an accident. The more products it sells, the higher the probability of at least one accident and thus the higher the probability that its inadequate attention to safety will be detected. Similarly, if an accounting firm is not rigorous in an audit of a single company in a single year, the probability that its lack of rigor will become known is less than one. Nevertheless, if an accounting firm is consistently sloppy in its audits of many companies, the low quality of its audits will come to light eventually.

The same assumption – that bad quality is detected with probability less than one and that the probability of detection is positively correlated with the frequency of repeat play – is also sufficient to generate reputational economies of scale in a finite-horizon model of reputation with two types similar to the one pioneered by Kreps and Wilson.[7]

Section II briefly sets out the point made by Rasmusen and Iaccobucci that, under the basic infinitely repeated game model of reputation, there are no reputational economies of scale. Section III modifies the basic model by assuming that the probability of detection is less than one. It shows that the minimum quality-assuring price decreases with the volume of sales. Section IV extends the analysis to the situation where firms are of different sizes. Because the quality-assuring price is lower for larger firms, firms which sell more will force smaller competitors out of the market. Section V analyzes the umbrella branding context and shows that a trademark owner can leverage its trademark from one competitive market to another and that doing so may make the market for high quality viable. Section VI analyzes reputational economies of scale under a finite-horizon model with two types. Section VII discusses extensions, and Section VIII concludes.

II. Lack of Economies of Scale in the Basic Model

This section generally follows Eric Rasmusen's 1989 and 2007 formalization of Klein and Leffler's 1981 article.[8] There are an infinite number of potential firms and consumers. In each period, there are n firms that participate in the market by selling at least one unit of the relevant good. Each participating firm incurs a fixed cost, $F>0$, which is paid at the beginning of the first period in which it participates. A firm that chooses not to participate gets payoff of zero. Each period, each firm can choose to make products of high or low quality. Quality cannot be

[7] Id.

[8] Eric Rasmusen, A Simple Model of Product Quality with Elastic Demand, 29 Economics Letters 281 (1989); Eric Rasmusen, Games and Information (4th ed. 2007), pp. 137-40; Benjamin Klein & Keith Leffler, The Role of Market Forces in Assuring Contractual Performance, 89 Journal of Political Economy 615 (1981).

observed by consumers at the time of purchase. It costs a firm $c > 0$, paid at the end of the relevant period, to produce a unit of the high quality good; it costs zero to produce the low quality good. Each period, each firm chooses its price *p*. Each period, consumers decide how much of the product to buy from each firm. The amount consumers buy in each period from firm *i* is denoted q_i, and firms receive the payment at the end of the period. After purchasing the good, consumers learn the quality of the goods they purchased and can use that knowledge to determine which firm to purchase from in the next period. Knowledge is shared among all consumers. The payoff to a consumer of buying a low quality product is zero. The payoff to buying a high quality product varies among consumers, but for every price, there are a sufficient number of consumers willing to pay it to support several competing firms, although, of course, total demand, $q(p) = \sum_{i=1}^{n} q_i > 0$, falls with price, $\frac{dq}{dp} < 0$, where total demand is a differentiable and thus continuous function of price. The discount rate is $r>0$. The game repeats infinitely. Since this is an infinitely repeated game, there are many equilibria. The equilibria of interest, however, are those that sustain the production of high quality goods. The exposition below is confined to ascertaining the conditions for such equilibria.

Consider a firm that is deciding between two alternatives: (1) producing high quality products in every period and receiving a high price, and (2) producing low quality in every period, getting the high price in the first period and then zero profits in every other period. p^* is the price at which a firm would be indifferent between these two alternatives:

$$\frac{q_i(p^*-c)}{r} = \frac{q_i p^*}{1+r} \tag{1}$$

A little algebra shows that:

$$p^* = (1+r)c \tag{2}$$

This p^* is the quality-assuring price. It is the minimum price that gives a firm an incentive to produce high quality goods. Note that the quality-assuring price does not vary with the quantity produced by each firm, q_i.

It is a perfect equilibrium (a) for consumers to purchase randomly in the first period from firms selling at p^*, but, after that, to purchase randomly only from firms that sell at p^* and that have never sold a low quality product, and (b) for firms to produce high quality and sell at p^*. To make this price consistent with free entry, the profit for each firm must be zero. That is,

$$F = \frac{q_i(p^*-c)}{r} \tag{3}$$

Substituting $p^*=(1+r)c$, we can derive the equilibrium quantity produced by each firm, q_i^*

$$q_i^* = \frac{F}{c} \tag{4}$$

The equilibrium number of firms, n^*, entering the market in each period is derived by setting supply equal to demand:

$$n^* q_i^* = q(p^*) \tag{5}$$

Combining (3), (4) and (5):

$$n^* = \frac{cq(p^*)}{F} = \frac{cq((1+r)c)}{F} \tag{6}$$

where $q((1+r)c)$ denotes q, total quantity, as a function of $(1+r)c$, not q times $(1+r)c$. The most important point is that the equilibrium price in equation (2) does not vary with quantity.[9] Suppose, for example, that a firm were somehow able to double its quantity from q_i to $2q_i$. The quality-assuring price would still be given by the price that makes the firm indifferent between high quality and low quality. Modifying equality (1) to take into account the doubling of the quantity:

$$\frac{2q_i(p^* - c)}{r} = \frac{2q_i p^*}{1+r} \tag{7}$$

Doubling the quantity both doubles the benefit of faithfully producing high quality -- the left-hand side of equation (7) – and doubles the benefit of cheating -- the right-hand side of equation (7) . As a result, increasing quantity has no effect -- the 2s cancel each other out. So equation (7) easily reduces to equation (1), and the quality-assuring price in (2) remains the same regardless of the quantity produced by each firm. This is the essence of the argument made by Iacobucci (2012) on p. 310. In slightly more complicated form, it is the argument made by Rasmusen (2016) on pp. 267-28.

III. Reputational Economies of Scale When Low Quality Is Detected With Probability Less Than One

A key assumption in the prior section was that consumers detect low quality with probability one at the end of each period. Suppose the probability with which the low quality of any particular unit purchased is detected is ρ, $0 < \rho < 1$ and independent. Let s be the probability that low quality is detected in at least one unit produced by a firm in a given period. If q_i units are sold, $s = 1 - (1-\rho)^{q_i}$. The probability of detection, s, obviously increases with quantity, q_i. The equation for the quality-assuring price – see (1) above -- needs to be modified to take into account that cheating is discovered with probability less than one. Let p'^* denote the quality-assuring price under the assumption that the probability of detection is less than one. The payoff of consistently producing high quality remains the same, but the payoff to producing low quality

[9] Integer problem.

is more complicated, because a firm producing low quality goods may now get the high payoff for several periods until the low quality of its products is detected. As before, the quality-assuring price, p'^*, is determined by setting the payoff for producing high quality in every period equal to the payoff for producing low quality in every period, getting the high price in the first period or periods and then zero profits in every other period: [10]

$$\frac{q_i(p'^*-c)}{r} = \frac{q_i p'^*}{1+r} + (1-S)\frac{q_i p'^*}{(1+r)^2} + (1-s)^2\frac{q_i p'^*}{(1+r)^3} + \cdots \qquad (7)$$

$$= \sum_{j=1}^{\infty}(1-s)^{j-1}\frac{q_i p'^*}{(1+r)^j}$$

$$= \frac{q_i p'^*}{(1+r)}\sum_{j=0}^{\infty}\left(\frac{1-s}{1+r}\right)^j$$

$$= \frac{q_i p'^*}{(1+r)}\left(\frac{1}{1-\left(\frac{1-s}{1+r}\right)}\right)$$

$$= \frac{q_i p'^*}{r+s}$$

Solving for the quality-assuring price, p'^*:

$$p'^* = \frac{(r+s)c}{s} \qquad (8)$$

As one would expect, as the probability of detection, s, goes to one, p'^* in (8) converges to p^* in (2). Note, however, that since $s = 1-(1-\rho)^{q_i}$, the quality-assuring price now varies with the quantity produced by each firm, q_i. In particular, as q_i approaches infinity, s approaches one, and price approaches (1+r)c. On the other hand, as q_i approaches zero, s approaches 0, and p'^* approaches infinity. It is relatively easy to see that p'^* is a strictly monotonically decreasing

[10] Although the literature often assumes that firms are constrained to offering either all high-quality or all low-quality, it is useful to check that the quality assuring price does not give firms an incentive to produce some high quality and some low quality product. After all, this and most other models in the literature assume fixed marginal cost, so there is no loss in economies of scale if a firm chose to produce some high quality and some low quality. Appendix B proves that, under the quality assuring price in Equation (8), a firm rationally produces all high quality rather than a mixture of qualities. This result could also by justified if one assumed that producing the fixed cost, F, was incurred for each quality, e.g. if each quality required a different factory and/or separate management structure.

function of q_i. This makes intuitive sense. When quantity is low, the probability of detection is low, so the firm needs a high price to make it worthwhile not to cheat. This is closely related to the standard result in the economic analysis of crime, where optimal sanctions increase as the probability of detection goes down. In the reputation model, the "sanction" for cheating is loss of the rents produced by receiving the high price in every period. On the other hand, as the probability of detection increases, the firm finds it profitable to produce high quality even if the price is lower.

This is the key result of the paper. Since the quality-assuring price for a firm goes down as the firm produces higher quantities, the larger firm has an advantage. It can sell at a lower price and still have an incentive to produce high quality. This is the meaning of reputational economies of scale.

As with the basic model, there are many equilibria, but the equilibrium of interest is the one which sustains the production of high quality goods. It is a perfect equilibrium (a) for consumers to purchase randomly in the first period from firms that sell at p'^*, but, after that, to purchase randomly only from firms that sell at p'^* and that have never been detecting as selling a low quality product, and (b) for firms to produce high quality and sell at p'^*. To make this price consistent with free entry, the profit for each firm must be zero. That is, the present discounted value of rents must equal the fixed cost of entry.

$$F = \frac{q_i(p'^* - c)}{r} \qquad (9)$$

Substituting $p'^* = \frac{(r+s)c}{s}$, we can derive the equilibrium quantity, $q_i'^*$

$$\frac{q_i'^*}{s} = \frac{F}{c}$$

$$\frac{q_i'^*}{1-(1-\rho)^{q_i'^*}} = \frac{F}{c} \qquad (10)$$

Again, it is instructive to compare equation (10) with the equation (4), the equivalent expression when the probability of detection equals one. As one would expect, as the probability that each low quality product will be detected, ρ, approaches one, equation (10) converges to equation (4). Similarly, as quantity, $q_i'^*$, approaches infinity, so the probability that low quality among any product will be detected approaches one, equation (10) also approaches equation (4). When $q_i'^*<1$, the left side of equation (10) can behave strangely, but quantities below one can be safely ignored as economically irrelevant. When $q_i'^*=1$, the left-hand side of equation (10) is $1/\rho$, where ρ is the probability that low quality is detected for each unit. As q_i increases from one, the left-hand side increases monotonically and approaches infinity. This means that, as long as $\rho > c/F$, there exists a quantity, $q_i'^*$, such that equation (10) is satisfied. The condition, $\rho > c/F$, means that there will not be an equilibrium in situations where either the probability that low quality will be detected, ρ, is very low, or where fixed costs are very low in relation to the variable costs of producing high quality. These conditions make sense. For reputation and repeat play to work, there has to be some decent probability that low quality will be detected; otherwise, cheaters won't be detected with sufficient probability to make high quality worthwhile. Similarly, if fixed costs are very low in relation to variable costs, then rents would need to be low to assure zero profits, and such low rents would not be consistent with the need to keep prices up to induce high quality.

Given that $q_i'^*$ exists for ρ >c/F, it is trivial to find the equilibrium number of firms, n'^*, by finding the value that satisfies:

$$n'^* q_i'^* = q(p) \tag{11}$$

$$n'^* = \frac{q(p'^*)}{q_i'^*} \tag{12}$$

It should be noted that the equilibrium described in this section is informationally demanding. Both consumers and producers need to know the quality-assuring price, and this requires that both consumers and producers know the cost of producing high quality goods, producers' fixed costs, and the probability that low quality will be detected. The need for the first piece of information is a characteristic of the basic infinitely repeated game model of reputation. The need for information on fixed costs and the probability that low quality is detected is, however, an additional requirement of the modified model presented in this section. Rasmusen (2001) suggests using a more complicated model, that it might be possible to relax some of these informational requirements.[11] Further research could explore the extent to which the results in this section are robust to parties estimating the parameters with error. In the real world, consumers do seem to have some sense that certain low prices are "too good to be true." This suggests that the model's prediction that consumers would refuse to buy from producers who charged too low a price has some plausibility. The basic and modified models are also informationally demanding in requiring all consumers to know when a firm has sold a single low quality product to any consumer in a prior period. This is, of course, unrealistic, but could probably be addressed relatively easily at the expense of complicating the model. Similarly, both the basic and modified model assume that low quality never happens through chance. Again, a more realistic but more complex model would assume that investments in quality reduce the probability of low quality output, but do not reduce that probability to zero.

IV. Different Sized Firms

[11] Eric Rasmusen and Timothy Perri, "Can High Prices Ensure Product Quality When Buyers Do Not Know the Sellers' Cost?" 39 Economic Inquiry 561 (2001).

In the model in the prior section, firm size (the quantity sold by each firm) was endogenous, and, in equilibrium, all firms were of the same size. Suppose, however, some firms are able to sell more than others. For example, perhaps, in addition to individual consumers there are a few bulk corporate or governmental buyers who buy on long-term contracts.[12] Even if the bulk buyers chose from whom to purchase randomly, those firms would acquire a price advantage, because, as noted in the prior section, the quality-assuring price falls with quantity. The larger firms with the bulk contracts would underprice the other firms in the individual market and force the smaller firms out of the market. Similar size difference could emerge if individual consumers were influenced to prefer the products of some firms over others, perhaps through advertising, favorable press, or other factors.

V. Umbrella Branding

Now consider what would happen if some firms sell multiple goods while other sell only one good. For simplicity, suppose there are two goods, A and B, and three types of firms, firms that produce only A, firms that produce only B, and firms that produce both A and B. This section will show that firms that produce both A and B will be able to sell both A and B at lower prices than firms which produce only A or only B. As a result, firms which produce only A or B will not survive in equilibrium, and only those that produce both A and B will survive. This establishes the core idea of reputational economies of scale – larger firms which sell more products have a competitive advantage over smaller ones. The intuition for this result is the same as for the result in the previous section. The firm that sells both goods sells more total goods, so, because the quality assuring price falls with quantity, the firm selling both goods will be able to sell at a lower cost and drive single-good firms out of the market. Nevertheless, the math demonstrating this result gets rather complicated, because one must take into account the price, quantity, and the number of firms in two different markets.

The analysis in this section depends crucially on how consumers react to the detection of low quality in one product produced by a firm that produces multiple products. If a consumer purchases product A and it turns out to be of low quality, the consumer might cease purchasing product A from that firm, but still purchase product B from it. Or, such a consumer might shun all products from that firm, that is, avoid purchasing both A and B from it. If consumers behave in the former way – treating low quality in one product as irrelevant to the quality of other products produced by the same firm – then there is no advantage to the two-product firm, and the equilibrium with different sized firms is the same as if firms were all the same size. On the other hand, if, as seem plausible, consumers interpret low quality in one product to mean that the firm is cutting corners on both products, then it makes sense for them to refuse to purchase A or B from that firm. If so, a new and interesting equilibrium arises. That is the equilibrium which will be analyzed in the rest of this section. This equilibrium is plausible in situations where a

[12] To preserve the incentive to produce high quality goods, these contracts would need to give the bulk buyers the right to cancel the contract without penalty if any buyer detected low quality.

producer has chosen to market two or more products under the same trademark, -- for example several different car models under the trademark "Toyota" or several types of toothpaste under the trademark "Colgate." In these situations, it is plausible that consumers assume that quality standards are similar for all products marketed under the same "umbrella trademark."

Notation needs to be modified to reflect that there are now two different goods. Denote the quantity of good A produced by firm i as q_{Ai} and the quantity of good B produced by such a firm is q_{Bi}, and denote the cost of producing high quality of each good as $c_A > 0$, and $c_B > 0$. Denote the probability with which the low quality of any particular unit purchased is detected as ρ_A and ρ_B, where these two probabilities are independent. Let $s_A = 1 - (1 - \rho_A)^{q_{Ai}}$ be the probability that low quality is detected in at least one unit produced by a firm, if the firm sells low quality of product A, but either no product B or only high quality of product B. Let $s_B = 1 - (1 - \rho_B)^{q_{Bi}}$ be the probability that low quality is detected in at least one unit produced by a firm, if the firm sells low quality of product B, but either no product A or only high quality of product A. Let s_{AB} be the probability that low quality is detected in at least one unit produced by the firm if the firm produces low quality of both A and B, $s_{AB} = s_A + s_B - s_A s_B = 1 - (1 - \rho_A)^{q_{Ai}}(1 - \rho_B)^{q_{Bi}}$. Note that, holding quantities of A and B constant, $S_{AB} > S_A$ and $S_{AB} > S_B$. This makes sense, because if two firms produce the same number of low quality goods of kind A, and one of them also produces low quality goods of kind B, it is more likely that the firm which produces two types of low quality goods will be caught. After all, the probability that the two-good firm is caught making low quality goods of kind A is the same as the probability that the one good firm is caught making low quality goods of kind A, but the two-good firm also has some probability of being caught making low quality goods of kind B. Let $F_A > 0$, and $F_B > 0$ be the fixed costs of producing A and B respectively. As in the one-good case, these costs are paid at the beginning of the first period in which the firm produces the relevant good. Note that F_A does not depend on whether the firm produces one or two goods, nor does F_B. That is, there are no economies of scope attributable to the fixed costs. This assumption, like the assumption of constant marginal costs, allows the analysis to focus economies scale created by reputation rather than by technological factors. As in the one good case, the payoff to buying high quality of product A or B varies among consumers, but for every price and each good, there are a sufficient number of consumers willing to pay it to support several competing firms, although, of course, total demand, $q_A(p) = \sum_{i=1}^{n} q_{Ai} > 0$ and $q_B(p) = \sum_{i=1}^{n} q_{Bi} > 0$, falls with price, $\frac{dq_A}{dp} < 0$ and $\frac{dq_B}{dp} < 0$, where total demand for A and B is a differentiable and thus continuous function of the price of each good.

The goal of this section is to find quality assuring prices for the two good firm which are lower than those for the single good firm and consistent with competition. For this to be true, the following six conditions (C1-C6) must be satisfied

C1. High quality on both A and B at least as profitable as low quality on both A and B:

$$\frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)}{r} \geq \frac{q_{Ai}p_A'^{*}+q_{Bi}p_B'^{*}}{(r+s_{AB})}$$

C2. High quality on both A and B at least as profitable as high quality on B and low quality on A:

$$\frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)}{r} \geq \frac{q_{Ai}p_A'^{*}+q_{Bi}(p_B'^{*}-c_B)}{(r+s_A)}$$

C3. High quality on both A and B at least as profitable as high quality on A and low quality on B:

$$\frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)}{r} \geq \frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}p_B'^{*}}{(r+s_B)}$$

C4. Prices are lower for a firm producing both A and B than for a firm selling only A or only B (assuming equal quantities):

$$p_A'^{*} < \frac{(r+s_A)c_A}{s_A} \quad \text{and} \quad p_B'^{*} < \frac{(r+s_B)c_B}{s_B}$$

C5. Competition drives down prices to the lowest values consistent with C1-C3:

Either C1 or C2 or C3 holds with equality

C6. New firms enter the market until profits drop to zero:

$$F_A + F_B = \frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)}{r}$$

The existence of an ordered pair, $(p_A'^{*}, p_B'^{*})$, that satisfies these constraints is proved in Appendix A. Appendix B shows that it would not be rational for a firm to produce some high and some low quality of A and/or some high and some low quality of B.

Note that if the quantity of A produced by the A-only firm and by the firm producing both A and B are equal, then the quality-assuring price will be lower for the firm producing both A and B than for the firm producing only A. This price advantage directly follows from the discussion in Section III of expression (8) where it was shown that the quality-assuring price for a firm goes down as the firm produces higher quantities. Here, the firm produces both A and B, and thus produces at higher quantities than the firm that produces only A. Similarly, the quality-assuring price for the two product firm will be lower than for a firm producing only good B. From the two-product firm's price advantage, it is easy to see that the two-product firm will dominate the market for both A and B. Consider the following set of strategies which form a perfect equilibrium.

Let $p_A'^*$ and $p_B'^*$ be the pair of prices identified in Appendix A as satisfying constraints C1-C6.

(1) Consumers purchase good A randomly in the first period from firms that sell both A and B and that sell good A at $p_A'^*$. After the first period, consumers purchase good A randomly from firms that sell both A and B and that sell good A at $p_A'^*$ and that have never been detected as having sold a low quality product.

(2) Consumers purchase good B randomly in the first period from firms that sell both A and B and that sell good B at $p_B'^*$. After the first period, consumers purchase good B randomly from firms that sell both A and B and that sell good B at $p_B'^*$ and which have never been detected as having sold a low quality product.

(3) Firms that produce both A and B produce both at high quality and sell them at prices $p_A'^*$ and at $p_B'^*$ respectively.

(4) Firms that would produce only A or only B never enter the market.

Note that it is rational for consumers to purchase only from firms which sell both A and B, because a firm that produced only A or only B would not rationally produce high quality goods at $p_A'^*$ or $p_B'^*$. For example, if a firm that produced only good A sold its goods at $p_A'^*$, it would have no incentive to produce high quality, because, as pointed out above, $p_A'^*$ is lower than the quality-assuring price for the A-only firm. So it would produce low quality. So no rational consumer would purchase from it at that price. Since consumers won't purchase from a firm that produced only good A or good B, such firms rationally won't pay the fixed cost to enter the market.

Note, of course, that there are many equilibria, including equilibria in which both two-product and single-product produce goods of similar quality. The most obvious (and uninteresting) is the equilibrium in which consumers purchase only products sold at the low quality price (here normalized to zero) and in which all producers produce only low quality. A more interesting equilibrium is one in which consumers favor single-product firms by buying from them with higher probability, even if the single product firm offers the good at the same price as the two-product firm. By doing so, consumers could negate the scale advantage of the two-product firm by purchasing, in aggregate, more of the single good from the single-product firm. For example, suppose goods A and B are similar in that the cost of producing high quality is equal, $c_A=c_B$, the probability that low quality is detected is equal, $\rho_A=\rho_B$, and demand for the two products is the same. Under those assumptions, there would be an equilibrium in which each consumer purchased randomly from all firms, but in which the probability with which the consumer purchased from each single-good firm was twice the probability with which the consumer purchased from each two-good firm. If consumers behaved in this fashion, each two-product firm would sell half as much of each good as each single-product firm. So the total sales (A and B combined) of the two-product firms would equal the sales of the single-product firm. So the quality-assuring price for both products would be the same for two-product and one-product firms. While interesting, this equilibrium assumes implausible consumer behavior.

So far this paper has assumed that consumer demand is structured so that, no matter the price, there is always sufficient demand to support multiple competing firms. As Rasmusen points out, however, it is possible that consumers will prefer low quality to high quality at the quality-assuring price that could be offered by a single-product firm.[13] The existence of multi-product firms can, in some circumstances, solve this problem by lowering the quality-assuring price. In this way, umbrella branding (the use of a single trademark for several products produced by the same firm) makes high quality viable in situations where, if the same product were produced by a single-product firm, high quality would require a quality-assuring price that consumers were unwilling to pay. Note, contrary to Rasumusen, umbrella branding makes high quality viable even in the presence of perfect competition.[14]

While this section focused on umbrella branding, the law firm size issue explored by Iacobucci[15] is analytically identical. Instead of a manufacturer making two goods, he considers a law firm producing two kinds of legal services, where the two services are distinguished by the fact that each is produced by a different lawyer. Nevertheless, the model would be the same. For each lawyer, there is a probability that low quality will be detected. If consumers punish both lawyers in a firm if they detect low quality by one lawyer, then low quality is likely to be punished more swiftly in a multi-member firm. This enables the multi-member firm to credibly offer high quality at a lower quality-assuring price than either lawyer could offer if she practiced alone.

VI. Reputational Economics of Scale in a Finite-Horizon Model

This section shows that reputational economies of scale exist also in a finite-horizon model similar to that pioneered by Kreps and Wilson.[16] In this model, there are two types of firms, good and bad. Firms know their types, but consumers do not know firm types. Good firms always produce high quality goods, even if doing so is not profitable, while bad firms are opportunistic and produce high quality only if it maximizes the present discounted value of their profits. The literature sometimes calls good firms “commitment type” firms, while bad firms are “strategic type” firms. With probability $\emptyset$, a firm is good, and with probability 1- $\emptyset$ a firm is bad.

[13] See supra n. 4.
[14] Id.
[15] Id.
[16] David Kreps & Robert Wilson, Reputation and Imperfect Information, 27 J. Econ Theory 253-79 (1982). For a similar model with an infinite number of types, see Paul Milgrom & John Roberts, Predation, Reputation and Entry Deterrence, 27 J. Econ Theory 280-312 (1982).

$0 < \emptyset < 1$. Assume that high quality goods cost *c* to produce, $0 < c < 1$, while low quality goods cost zero to produce. Consumer are willing to pay 1 for high quality goods and zero for low quality goods. If consumers are uncertain about the quality of goods, the price they are willing to pay is proportional to the probability that they think quality will be high. So, if consumers think only good firms will produce high quality, and they cannot distinguish between good and bad firms, they are willing to pay $\emptyset$. As in the main model, the probability with which the low quality of any particular unit purchased is detected is ρ, $0< \rho <1$, and independent, and *s* is the probability that low quality is detected in at least one unit produced by a firm in a given period. If q units are sold, $s = 1 - (1 - \rho)^q$. There is only one firm. The firm sets the price, and consumers decide whether or not to buy. The discount factor is δ, $0< \delta<1$. Consumers know all parameters -- $\emptyset$, *c*, ρ, *s*, and δ --, but consumers do not know what type a firm is. If a firm is detected having produced low quality, that fact is known to all consumers. For convenience, if the payoffs to high and low quality are the same, it is assumed that the bad firm produces high quality.

Consider first a one-period game. Obviously, the bad firm will produce low quality and the good firm will produce high quality. As a result, the price will be $\emptyset$.

Now consider the strategy of a bad firm in a two-period game. It will produce high quality if doing so maximizes the present discounted value of its profits. Assuming that consumers will believe it produces high quality in the first period (an assumption justified below), the bad firm will produce high quality if:

$$1 - c + \partial\emptyset \geq 1 + \partial(1 - s)\emptyset \quad (13)$$

The left side represents the bad firm's profits if it produces high quality in the first period and it is believed to produce high quality. It gets 1 in the first period (the price consumers are willing to pay for high quality), and it pays *c* to produce it. Because it produced high quality in the first period, consumers are willing to purchase from it in the second period. Nevertheless, because consumers know that the bad firm will produce low quality in the second (last) period, and since they do not know whether the firm is low quality, the price in the last period is $\emptyset$, as in the one-period game. Since it costs the bad firm nothing to produce low quality, its discounted profits in the second period are $\partial\emptyset$. If the bad firm produces low quality in the first period, it gets profits of 1 (the price of high quality minus the costs of production, which are zero). Low quality is detected with probability s, and consumers rationally do not purchase from bad firms in the last period, so the present-discounted second-period profits are $\partial(1 - s)\emptyset$. Rearranging the terms of expression (13), the bad firm produces high quality in the first period if $\emptyset \geq \frac{c}{\partial s}$. As in Kreps and Wilson's model, under some parameters, it is equilibrium behavior for bad firms to mimic good firms in all periods other than the last period, and it is rational for consumers to believe that bad firms, under some parameters, produce high quality, except in the last period.

If this inequality is satisfied, then it is a perfect Bayesian equilibrium for a bad firm to produce high quality in the first period (and to sell goods for 1 in the first period) and for a bad firm to sell low quality in the last period (and for the bad firm (and good firm) to sell its goods for $\emptyset$ in the second period). In this equilibrium, consumers purchase goods for a price of 1 in the first period, and purchase goods for $\emptyset$ in the second period, unless a firm has been detected as having sold low quality in the first period, in which case consumers refuse to buy from the firm in the second period (or buy only at price zero). If consumers in the first period do not buy at the prices stated above, the firm does not sell in the second period. If firms sell at prices other than those stated above, consumers do not buy anything. Appropriate beliefs can be constructed to make these off-equilibrium path behaviors rational.

Note the effect of *s*. As *s* goes up, the inequality in (13) is more likely to be satisfied, because *s* appears only on the right side, and the right side decreases as s increases. That is, the inequality is satisfied for lower values of $\emptyset$ and ∂ and for higher values of *c*. Since *s* is an increasing function of quantity, *q*, this means that consumers are more likely to trust large firms. Thus, as under the infinitely-repeated game model analyzed in prior sections, there are reputational economies of scale. Since, in this model, the sellers are assumed to have pricing power, the reputational advantage is not in offering lower prices, but rather that a bad firm is more likely to be trusted to produce high quality (and more likely to do so) over a wider range of parameters. The model could be easily modified to show a pricing advantage of the larger firm, because *c* can be thought of not as absolute cost, but rather as the ratio of cost to price. Thus, as *s* goes up, it is rational for a bad firm to produce high quality even at a lower price. So a larger firm will be able to underprice a smaller firm. That is, suppose consumers were willing to pay price, $p¸c < p < 1$, for high quality goods, then there are prices such that it would be an equilibrium for a larger bad firms to produce high quality for price *p*, but not for a smaller bad firm.

Now consider a three-period game. There are two cases to consider, where $\emptyset \geq \frac{c}{\partial s}$, so the bad firm can be assumed to produce high quality in the second period if low quality is not detected in the first period, and where $\emptyset < \frac{c}{\partial s}$, so the bad firm can be assumed to produce low quality in the second period. Consider first the situation where $\emptyset \geq \frac{c}{\partial s}$. An equilibrium in which the bad firm produces high quality in the first period is plausible if:

$$1 - c + \partial(1 - c) + \partial^2\emptyset \geq 1 + \partial(1 - s) + \partial^2(1 - s)^2\emptyset \quad (14)$$

The left side is the payoff if the bad firm produces high quality in the first and second periods and low quality in the third. The right hand side is the payoff if the bad firm produces low quality in all periods. It is assumed (and justified below) that consumers expect the firm to produce high quality in the first and second period and low quality in the third. Note that the expression above is more likely to be satisfied when *s* is high, because *s* appears only in the right-hand side, and higher values of *s* always makes the right-hand side smaller, thus increasing the range of parameters for which the inequality holds. Thus, as in the two-period game, there are reputational economies of scale, and consumers are more likely to trust large firms. It is, of course, necessary to check that it is rational for the firm to produce high quality in the first period. Solving (14) for $\emptyset$, dividing top and bottom ∂, and rearranging the numerator, expression (14) can be rewritten as:

$$\emptyset \geq \frac{c + \frac{c}{\delta} - s}{\partial(1 - (1 - s)^2)} \quad (15)$$

It is relatively easy to show that this expression is satisfied whenever $\emptyset \geq \frac{c}{\partial s}$, which is the assumption for this case, because the numerator of expression (15) is always smaller than c and the denominator of expression (15) is always larger than ∂s, so any $\emptyset \geq \frac{c}{\partial s}$ satisfies (15). Compare the numerator of the right-hand side (15) to the numerator of $\frac{c}{\partial s}$. That is, compare $c + \frac{c}{\delta} - s$ to c. The numerator of the former is always smaller, because $\frac{c}{\delta} - s < 0$ whenever $1 > \emptyset \geq \frac{c}{\partial s}$, which is our assumption for this case together with the fact that $\emptyset$ is a proportion and so must be less than 1. Similarly, compare the denominator of the right-hand side of (15) to the denominator of $\frac{c}{\partial s}$. That is, compare $\partial(1 - (1 - s)^2)$ to ∂s. The former is always larger, because $0 < s < 1$, because s is a probability. Thus, a bad firm will provide high quality in the first period of the 3-period game if it would be rational for it to produce high quality in the second period.

Under this equilibrium, it is rational for consumers to believe that the bad firm will produce high quality in the first and second periods (and thus for consumers to pay 1) and that the bad firm will produce low quality in the third period (and thus for the consumers to pay $\emptyset$), as long as the consumer refuses to pay these prices from any firm that has been detected as producing low quality in the past. Appropriate actions and beliefs off the equilibrium path can be easily constructed.

Now consider the three-period game where $\emptyset < \frac{c}{\partial s}$, that is, where the bad firm will produce low quality in the second and third periods. As in Kreps and Wilson's model,[17] it is possible that the bad firm will produce high quality in the first period, even though it will produce low quality in later periods. Such an equilibrium would be plausible if:

$$1 - c + \partial\emptyset + \partial^2(1 - s)\emptyset \geq 1 + \partial(1 - s)\emptyset + \partial^2(1 - s)^2\emptyset \quad (16)$$

As before, the left side is the payoff if the bad firm produces high quality in the first period, but not in any other period, whereas the right side is the payoff if the bad firm produces low quality in all periods. It is assumed (and justified below) that consumers expect the firm to

[17] Id.

produce high quality in the first period and low quality in the second and third periods. First note that, as in every other case, as s gets larger, the inequality is more likely to hold. Differentiating the left side by s yields $-\partial^2\emptyset$. In contrast, differentiating just the second term on the right side by s yields $-\partial\emptyset$. Since $0 < \partial < 1$, and since differentiating the third term will yield an expression that is always negative for $0 < s < 1$, increasing s will reduce the right side more than the left-side, so the inequality will hold under a greater range of parameters as s increases, thus again demonstrating reputational economies of scale. Note also that expression (16) will sometimes hold even when $\emptyset < \frac{c}{\partial s}$. That is, even when the bad firm will not produce high quality in the second or third period, it may be rational for it to produce high quality in the first period. This is apparent by comparing expression (16) to expression (13), where expression (13) is the condition for producing high quality in the first period of the two-period game, and thus the condition that led to the conclusion that the bad firm will produce low quality in the second period of the three-period game when $\emptyset < \frac{c}{\partial s}$. All but the last term on each side of (16) are the same as the terms of (13). So, if $\emptyset = \frac{c}{\partial s}$, all but the last term on the left and right (16) will be equal. The last term on the left of (16) is always be larger than the last term on the right, because $0 < s < 1$. Thus, because these expressions are continuous, there exist values of $\emptyset < \frac{c}{\partial s}$ such that inequality (16) holds. Thus, as in Kreps & Wilson's model,[18] it is more likely that a bad firm will be have well (like a good firm) in the first period of the three-period game than in first period of the two-period game, just as it was more likely that a bad firm would produce high quality in the first period of the two-period game than in the last period. As in the case where $\emptyset \geq \frac{c}{\partial s}$, consumers are rational to believe that a bad firm will produce high quality in the first period when inequality 16 is satisfied, and low quality in later periods, because it is equilibrium behavior for bad firms to do so. Of course, if inequality (16) is violated, the bad firm will produce low quality in all periods and equilibrium prices will fall accordingly.

Appendix C generalizes these results to games of any number of periods.

VII. Extensions

The models in this paper are, like all models, unrealistic in some respects. A key way in which these models are unrealistic is that they assume that if one consumer detects bad quality, all consumers become aware that the firm has produced bad quality. Nevertheless, if one assumed a more realistic diffusion of information, that would probably accentuate the reputational advantage of larger firms. The media are more likely to report on product defects that affect large numbers of consumers, so information about bad quality by large firms is likely to spread much more widely than bad quality by smaller firms. According to the logic of this paper, that will benefit the larger firms, because consumers can more reliably infer good quality from the absence of reports of bad quality. In the logic of the Becker deterrence model, large

[18] Id.

firms are more likely to be punished by the media for poor quality, and, as a result, the reputational bond they forfeit can be lower. Since the reputation bond is the degree to which prices are higher than marginal cost, that means that the larger firm can sell goods at a lower price and still have an incentive to maintain quality.

The model also assumes that if the firm chooses to produce with high quality, no low quality goods are produced. That is unrealistic, because even the best quality control cannot prevent production of an occasional defective product. Relaxing this assumption will again reinforce the reputational advantage of larger firms, because consumers can more easily discern whether defects are endemic or idiosyncratic when the firm produces a large number of goods. For example, if a firm produces ten goods and one is of poor quality, consumers cannot infer with confidence that the firm has bad quality control, because it is possible that the one good of poor quality reflects simply bad luck from a firm that produces high quality goods with probability much higher than 90%. On the other hand, if a firm produces one million goods and one hundred thousand are defective, the consumer can very reliably infer that the firm has poor quality controls that result in a high (10%) rate of defects.[19]

VIII. Conclusion

Relaxing the assumption that low quality is detected with certainty at the end of each period helps explain the widely assumed phenomenon of reputational economies of scale. Once this assumption is relaxed, reputational economies of scale emerge under the infinitely repeated game model of reputation, even in competitive markets and even if no assumptions are made about the order in which products or services are sold. Reputational economies of scale also occur in a finite-horizon game model of reputation with two types. Reputational economies of scale help explain many market phenomena, including gatekeeper liability, one-sided consumer contracts, umbrella branding, and the large size of firms in industries where product quality is hard to enforce through inspection or contract.

[19] The author thanks Steve Shavell for making the point in this paragraph.

Appendix A. Proof of the Existence of Quality-Assuring Umbrella Prices

As noted in the text, assume that total demand, $q_A(p) = \sum_{i=1}^{n} q_{Ai} > 0$ and $q_B(p) = \sum_{i=1}^{n} q_{Bi} > 0$, falls with price, $\frac{dq_A}{dp} < 0$ and $\frac{dq_B}{dp} < 0$, where total demand for A and B is sufficiently large to support competing firms and is a differentiable and thus continuous function of the price of each good. As in the text, also assume $r > 0$, $c_A > 0$, $c_B > 0$ $F_A > 0$, and $F_B > 0$.

Lemma 1. If firms producing both A and B choose prices and quantities that would be quality-assuring for firms producing just A or just B, then constraints C1, C2, and C3 would be satisfied with strict inequalities. That is, if $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$, then C1, C2 and C3 would hold with strict inequalities.

First consider C1. To Prove:

$$\frac{q_{Ai}(p_A'^* - c_A) + q_{Bi}(p_B'^* - c_B)}{r} > \frac{q_{Ai}p_A'^* + q_{Bi}p_B'^*}{(r+s_{AB})}$$

Substitute $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$ and simplify:

$$\frac{q_{Ai}\left(\frac{(r+s_A)c_A}{s_A} - c_A\right) + q_{Bi}\left(\frac{(r+s_B)c_B}{s_B} - c_B\right)}{r} > \frac{q_{Ai}\frac{(r+s_A)c_A}{s_A} + q_{Bi}\frac{(r+s_B)c_B}{s_B}}{(r+s_{AB})}$$

$$\frac{q_{Ai}c_A}{s_A} + \frac{q_{Bi}c_B}{s_B} > \frac{q_{Ai}c_A}{s_A}\left(\frac{r+s_A}{r+s_{AB}}\right) + \frac{q_{Bi}c_B}{s_B}\left(\frac{r+s_B}{r+s_{AB}}\right)$$

$s_{AB} > s_A > 0$ and $s_{AB} > s_B > 0$ (see p. 9), so the above inequality always holds.

Next consider C2: To Prove:

$$\frac{q_{Ai}(p_A'^* - c_A) + q_{Bi}(p_B'^* - c_B)}{r} > \frac{q_{Ai}p_A'^* + q_{Bi}(p_B'^* - c_B)}{(r+s_A)}$$

As with C1, substitute $p_A'^* = \frac{(r+s_A)c_A}{s_A}$ and $p_B'^* = \frac{(r+s_B)c_B}{s_B}$ and simplify:

$$\frac{q_{Ai}c_A}{s_A} + \frac{q_{Bi}c_B}{s_B} > \frac{q_{Ai}c_A}{s_A} + \frac{q_{Bi}c_B}{s_B}\left(\frac{r}{r+s_A}\right)$$

$r > 0$ and $s_A > 0$, so the above inequality always holds.

Similar reasoning shows that C3 also holds with strict inequality. Q.E.D.

Now consider the main proposition to be proved, that there exist prices, $(p_A'^*, p_B'^*)$, satisfying constraints C1 through C6. Consider ordered pairs, (p_A, p_B) and, in particular the set of ordered pairs constituting the line segment between $(c_{A,} c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$. The first point is defined by the marginal cost of producing each good, and the second point is defined by the quality-assuring prices when a firm produces only good A or only good B. Note that at $(c_{A,} c_B)$, C1, C2, and C3 will be each violated, because the left-hand sides will be zero and the right-hand sides will be positive. Note also that, according to Lemma 1, at $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C1, C2 and C3 each hold with strict inequalities.

Now consider what happens to C1 as one moves along the line from $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ to $(c_{A,} c_B)$ while setting the number of firms, *n*, such that C6 (zero profits) holds. For this purpose, it is helpful to rewrite C1 with total quantities rather than quantities per firm by multiplying the left and right-hand side of the inequality by the number of firms:

$$\frac{q_A(p_A'^* - c_A) + q_B(p_B'^* - c_B)}{r} \geq \frac{q_A p_A'^* + q_B p_B'^*}{(r + s_{AB})} \text{ (A1)}$$

For each set of prices on line segment between $(c_{A,} c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, we can find a number of firms, *n*, such that C6 (zero profits) holds because of continuity. The right-hand side of C6 is a continuous function of *n*, because the right-hand side of C6 is just the left hand side of (A1) divided by *n*. We have assumed that total demand for A and B is sufficiently large to support competing firms, so there is some value of $n \geq 2$ for which the right-hand side of C6 at equals or exceeds the left-hand side. We also know that as *n* goes to infinity, the value of the right-hand side of C6 goes to zero. So there must be some value of *n* for which the equality in C6 is true.

Note that since total demand for A and total demand for B, q_A and q_B, are assumed to be continuous functions of the prices, (p_A, p_B), and since s_{AB} is strictly positive and a continuous function of q_A and q_B, which in turn are continuous functions of (p_A, p_B), then both sides of the inequality (A1) must also be continuous functions of prices.

Since the inequality in (A1) holds strictly at $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ and is violated at $(c_{A,} c_B)$, and since the both sides vary continuously with p_A and p_B, there must be a place on the line between those two points where the left-hand and right-hand sides are equal. If there is only one such point, call that point (x_1, y_1). If there is more than one such point, denote as (x_1, y_1) the one with the highest value of p_A. Note that, for all points on the line between (x_1, y_1) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C1 holds with strict inequality, because if it held with equality that would violate the definition of (x_1, y_1), and if the inequality did not hold at all, then, because of continuity, there would have to be another point in that interval where C1 held with equality, which would also violate the definition of (x_1, y_1).

Now consider what happens to C2 as one moves along the line from $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ to $(c_{A,}c_B)$ while also assuming that C6 (zero profits) holds. Following reasoning similar to that in the prior paragraph, there will be one or more points on the line segment between $(c_{A,}c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ where C2 holds with equality. If there is only one such point, call that point (x_2, y_2). If there is more than one such point, denote as (x_2, y_2) the one with the highest value of p_A. For every point on the line between (x_2, y_2) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, C2 holds with strict inequality.

By similar reasoning, let (x_3, y_3) be the sole point where C3 holds with equality on the line segment between $(c_{A,}c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$ or the one with the highest value of p_A. C3 will hold with strict inequality for every point (x_3, y_3) and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$.

Let $(p_A'^*, p_B'^*) = \begin{cases} (x_1, y_1) \text{ if } x_1 \geq x_2 \text{ and } x_1 \geq x_3 \\ (x_2, y_2) \text{ if } x_2 > x_1 \text{ and } x_2 \geq x_3 \\ (x_3, y_3) \text{ if } x_3 > x_1 \text{ and } x_3 > x_2 \end{cases}$

By construction, all 6 constraints – C1 through C6 -- are satisfied at these prices, so there exists at least one pair of quality-assuring prices consistent with competition in which the two-good firms have prices lower than the one-good firms. Q.E.D.

Note that there are an infinite number of ordered pairs $(p_A'^*, p_B'^*)$, satisfying constraints C1 through C6. Their existence can be proved by considering the family of curves (e.g. increasingly concave or convex bowed-out lines) passing through $(c_{A,}c_B)$ and $\left(\frac{(r+s_A)c_A}{s_A}, \frac{(r+s_B)c_B}{s_B}\right)$, but overlapping or intersecting the straight line between those points only at the endpoints. By reasoning similar to the above for straight lines, each of those curves will also have a point where all six constraints are satisfied, and that point will not be on the straight line. For the purposes of this paper, it is sufficient to prove that there is just one such ordered pairs. Consideration of additional pairs satisfying the constraints introduces complications, such as how consumers coordinate on one pair of prices. By just considering the pair whose existence was proved first, such complications can be avoided.

Appendix B. No Mixing of High and Low Quality

The main text (and other articles in the literature) assume that a manufacturer that produces a single good produces either all high quality or all low quality of that good. Similarly, it is generally assumed that a manufacturer who produces two goods, A and B, produces uniform quality of A and uniform quality of B. That is, the text and literature assume that, although a firm may produce high quality A and low quality B, or low quality A and high quality B, the firm does *not* produce some high quality A and some low quality A and/or some high quality B and some low quality B. Given the assumption of fixed costs, *F*, that assumption may be reasonable. Perhaps producing some high and some low quality would require duplication of the fixed costs (or at least additional fixed costs). On the other hand, given the assumption that the marginal cost, *c*, is constant, no matter how many units are produced, it is also reasonable to consider the possibility that the manufacturer would choose to produce some high and some low quality goods.

First consider the Umbrella branding case, where there are two goods. Let k_A and k_B be the number of low quality units of A and B respectively, where $0 \leq k_A \leq q_{Ai}$, $0 \leq k_B \leq q_{Bi}$. One must prove the following inequality:

$$\frac{q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)}{r} \geq \frac{(q_{Ai}-k_A)(p_A'^{*}-c_A)+k_A p_A'^{*}+(q_{Bi}-k_B)(p_B'^{*}-c_B)+k_B p_B'^{*}}{r+\left(1-(1-\rho_A)^{k_A}(1-\rho_B)^{k_B}\right)} \text{ (A2)}$$

Note that if $k_A = k_B = 0$ that the left and right sides of the above expression would be equal. Note also that, given conditions C1, C2, and C3, if $k_A = q_{Ai}$ and/or $k_B = q_{Bi}$, the left and right sides of the above expression are either equal or the right side is smaller. The inequality can be rewritten as:

$$0 \geq r(k_A + c_A + k_B c_B) - (1-(1-\rho_A)^{k_A}(1-\rho_B)^{k_B})[q_{Ai}(p_A^{'*} - c_A) + q_{Bi}(p_B^{'*} - c_B)] \quad (A3)$$

Like A2, the right and left sides of A3 are equal when $k_A = k_B = 0$ and if $k_A = q_{Ai}$ and/or $k_B = q_{Bi}$, the left and right sides of the above expression are either equal or the right side is smaller. As a result, it is sufficient to prove that the second partial second-derivatives of the right-hand side with respect to k_A and k_B are positive. Since the expressions are identical with respect to k_A and k_B, it sufficient to prove the second derivative with respect to k_A is positive. The first derivative with respect to k_A is:

$$r + (1-\rho_A)^{k_A} ln(1-\rho_A)(1-\rho_B)^{k_B}[q_{Ai}(p_A^{'*} - c_A) + q_{Bi}(p_B^{'*} - c_B)]$$

The second derivative with respect to k_A is:

$$\left(1-\rho_A\right)^{k_A}\left(ln\left(1-\rho_A\right)\right)^2\left(1-\rho_B\right)^{k_B}[q_{Ai}(p_A'^{*}-c_A)+q_{Bi}(p_B'^{*}-c_B)]$$

This expression is the product of four terms, each of which is positive, so the whole expressions is positive. Q.E.D.

The one good case, where the manufacturer produces only one good, follows easily from the proof above. To prove that a firm would not produce some high and some low quality, just set $q_{Bi}=k_B=0$ and remove all the *A* subscripts. The proof then follows in exactly the same way.

Appendix C. Proof of Reputational Economies of Scale for Any Number of Periods in the Finite-Horizon Game

First, consider a game of n periods where $\emptyset \geq \frac{c}{\partial s}$. As shown in Section VI, this means that the bad firm will produce high quality in the first period of the two-period game, and in the first and second periods of the three-period game. The first goal of this section is to show that for any $n > 3$, the bad firm will produce high quality in all periods except the last. This will be rational if:

$$\left\{\sum_{i=1}^{n-1} \partial^{i-1}(1-c)\right\} + \partial^{n-1}\emptyset \geq \left\{\sum_{i=1}^{n-1} \partial^{i-1}(1-s)^{i-1}\right\} + \partial^{n-1}(1-s)^{n-1}\emptyset \qquad \text{(D1)}$$

Rearranging the terms, it is rational for the bad firm to produce high quality in all periods except the last if:

$$\emptyset \geq \frac{\sum_{i=1}^{n-1} \partial^{i-n+1}\left((1-s)^{i-1} - (1-c)\right)}{\partial(1-(1-s)^{n-1})} \qquad \text{(D2)}$$

Because we are assuming that $\emptyset \geq \frac{c}{\partial s}$, It is only necessary to prove that the right hand side of the inequality is less than $\frac{c}{\partial s}$:

$$\frac{\sum_{i=1}^{n-1} \partial^{i-n+1}\left((1-s)^{i-1} - (1-c)\right)}{\partial(1-(1-s)^{n-1})} \leq \frac{c}{\partial s} \text{ (D3)}$$

Proof by induction. Section VI already proves the inequality for $1 < n \leq 3$. Suppose D3 is true for $n = K > 3$:

$$\frac{\sum_{i=1}^{k-1} \partial^{i-k+1}\left((1-s)^{i-1} - (1-c)\right)}{\partial\left(1-(1-s)^{k-1}\right)} \leq \frac{c}{\partial s} \text{ (D4)}$$

We need only prove that D3 holds also for $n = K + 1 > 4$. That is, one must prove that:

$$\frac{\sum_{i=1}^{k} \partial^{i-k+1}\left((1-s)^{i-1} - (1-c)\right)}{\partial(1-(1-s)^{k})} \leq \frac{c}{\partial s}$$

The left side of the above expression can be rewritten as

$$\frac{\sum_{i=1}^{k-1} \partial^{i-k+1}\left((1-s)^{i-1} - (1-c)\right)}{\partial\left(1-(1-s)^{k}\right)} + \frac{(1-s)^{k-1} - (1-c)}{\left(1-(1-s)^{k}\right)} \text{ (D5)}$$

The first term differs from the first term of the induction assumption (D4) only because it has *k* rather than *k-1*, as the exponent in the denominator. That means that the denominator of D5 is larger, so the whole term is smaller than the first term of D4. This means that the first term of D5 is less than $\frac{c}{\partial s}$. The second term of D5 is always negative, because $\emptyset \geq \frac{c}{\partial s}$ and $0 < \emptyset < 1$ implies that $c < s$. If $c < s$, then $(1-s)-(1-c) < 0$, which implies that $(1-s)^{k-1}-(1-c) < 0$ as long as $K > 2$. Since Section VI already proved the inequality for $n \leq 3$, we need only consider $K > 2$. So the second term of D5 is always negative. As a result, because the first term is less than $\frac{c}{\partial s}$ and the second term is negative, the entirety of D5 must be less than $\frac{c}{\partial s}$. So, as long as $\emptyset \geq \frac{c}{\partial s}$, it will be rational for the bad firm to produce high quality in all periods except the last no matter how many periods there are.

Reputational economics of scale follow from inspection of D1. *s* appears only on the right-hand side, and the derivative of right-hand side with respect to *s* is negative for $0 < s < 1$. So, as s gets larger, the inequality holds under a wider array of parameters. As a result, the bad firm produces high quality for a wider array of parameters, thus making it trustworthy in a wider array of circumstances.

Now consider a game of *n* periods where $\emptyset < \frac{c}{\partial s}$. There are two possibilities. (1) that $\emptyset$ is such that in the game of $n-1$ periods the firm produced bad quality in all periods, (2) $\emptyset$ is such that in the game of $n-1$ periods the firm produced good quality in the first $m < n$ periods and produced bad quality only in the last $n-m$ periods.

Consider possibility (1) first, that $\emptyset$ is such that in the game of $n-1$ periods the firm produced bad quality in all periods. It is easy to show that the firm might produce high quality in the first period of the game of *n* periods. Let $\emptyset^*$ be the $\emptyset$ at which the firm in the game of $n-1$ periods would be indifferent between producing high and low quality in the first period and low quality in all other periods. The goal is to prove that for some $\emptyset < \emptyset^*$, it will be rational for the firm to produce high quality in the first period. The case of $n = 3$ periods has already been proven in Section VI. Assume that $n > 3$. By assumption:

$$1 - c + \sum_{i=1}^{n-1} \partial^i (1-s)^{i-1} \emptyset^* = 1 + \sum_{i=1}^{n-1} \partial^i (1-s)^i \emptyset^* (D6)$$

Now consider the condition for producing high quality in the first (and only the first) period of the game of $n+1$ periods:

$$1 - c + \sum_{i=1}^{n} \partial^i (1-s)^{i-1} \emptyset \geq 1 + \sum_{i=1}^{n} \partial^i (1-s)^i \emptyset \ (D7)$$

If $\emptyset = \emptyset^*$, the inequality will always hold, because the only difference between the left hand terms of D6 and D7 is that D7 has an additional, last term, $\partial^n(1-s)^{n-1}$. Similarly, the only difference between the right hand terms of D6 and D7 is that D7 has an additional last term, $\partial^n(1-s)^n$. Since the additional last term of left side of D7 is always larger than the additional last term of the right side, the inequality will always hold for $\emptyset = \emptyset^*$. Because everything is continuous, that also means that D7 will hold for some $\emptyset < \emptyset^*$. So, as the number of periods increases, it will be rational for the bad firm to produce high quality good in the first period over a larger range of parameters.

Note also that the range of parameters in which it is rational for firms to produce high quality goods will be larger when the firm produces more goods, so *s* is higher. As in Section VI, this is apparent by taking the first derivative of D6 with respect to *s*.

Now, still assuming $\emptyset < \frac{c}{\partial s}$, consider the second possibility, that $\emptyset$ is such that in the game of $n-1$ periods the firm produced good quality in the first $m < n$ periods and produced bad quality only in the last $n-m$ periods.

[I have not written up the rest of this proof yet.]