Tort Liability and Unawareness*

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Abstract

Unawareness is a form of bounded rationality in which a person fails to conceive all feasible acts or consequences or to perceive as feasible all conceivable act-consequence links. We study the implications of unawareness for tort law, where relevant examples include the discovery of a new product or technology (new act), of a new disease or injury (new consequence), or that a product can cause an injury (new link). We argue that negligence is superior to strict liability in a word with unawareness, because negligence, through the stipulation of due care standards, spreads awareness about the updated probability of harm. (*JEL* D83, K13)

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1 Introduction

A central question in the field of law and economics is whether negligence or strict liability is the more efficient tort liability rule. Under negligence, a victim can recover damages for harm caused by the activity of an injurer who failed to take reasonable care when engaging in the activity. Under strict liability, by contrast, a victim can recover damages for harm caused by the activity of an injurer irrespective of whether the injurer took reasonable care. The relative efficiency of the two rules is customarily measured by the Kaldor-Hicks criterion.

A bedrock result in the economic analysis of tort law is that, in the case of unilateral accidents with fixed activity levels, negligence and strict liability are equally efficient, provided that, in the case of negligence, the court properly sets the due care standard (the legal standard for what constitutes reasonable care) (Shavell, 1987).[1] This equivalence result, however, presents something of a puzzle in light of two facts about negligence. First, negligence is the dominant rule in Anglo-American law. Second, negligence is the more costly rule to administer, because the court must determine the due care standard and adjudicate whether the standard was met. The puzzle is that if negligence and strict liability are equally efficient but negligence is more costly to administer, why is negligence the dominant rule?

The negligence puzzle has led researchers to revisit the equivalence result by exploring departures from the standard accident model, which is based on the expected utility framework and the Bayesian paradigm. For instance, Teitelbaum (2007) and Chakravarty and Kelsey (2017) explore ambiguity (Knightian uncertainty). They assume that the relevant parties are Choquet expected utility maximizers with neo-additive beliefs about accident risk,[2] and they find that this breaks the equivalence in favor of negligence.

In this paper, we explore *unawareness*. Unawareness is the failure to conceive or perceive the entire state space. It is a form of bounded rationality in which a person fails to conceive

[1] In unilateral accidents, the injurer, but not the victim, can take care to reduce expected harm. In unilateral accidents with fixed activity levels, the injurer affects expected harm only through his level of care (and not through his level of activity). The equivalence result also holds in the case of bilateral accidents with fixed activity levels, provided that strict liability is coupled with the defense of contributory negligence.

[2] The neo-additive Choquet expected utility model was developed by Chateauneuf et al. (2007).

all available acts or potential consequences or fails to perceive as feasible all conceivable act-consequence links. Unawareness creates the possibility of *growing awareness*—the expansion of the state space when a person discovers a new act, consequence, or act-consequence link. Examples relevant to tort law include the discovery of a new product or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery that a known product can cause a known injury (new act-consequence link).

We study the implications of unawareness for tort law, and specifically for the negligence versus strict liability debate. To model unawareness and growing awareness, which requires a theory of how beliefs update as the state space expands, we adopt the *reverse Bayesianism* approach of Karni and Vierø (2013). Karni and Vierø posit that as a person becomes aware of new acts, consequences, or act-consequence links, his beliefs update in a way that preserves the relative likelihoods of events in the original state space. More specifically, they postulate that (i) in the case of a new act or consequence, probability mass shifts proportionally away from the events in the original state space to the new events in the expanded state space, and (ii) in the case of a new act-consequence link, null events in the original state space become nonnull, and probability mass shifts proportionally away from the original nonnull events to the original null events that become nonnull.[3]

We argue that negligence is superior to strict liability in a world with unawareness. Under either rule, a tort litigation involving a new act, consequence, or act-consequence link makes the world aware of a new possibility of harm. Under negligence, however, the litigation provides the world with more information. In particular, the court's stipulation of a new due care standard provides the world with information about the updated probability of harm. This information is necessary for either rule to induce the injurers of the world to take efficient care. Negligence provides this information to injurers. Under strict liability, they would have to expend additional resources to develop this information.

[TBA: A discussion of the related stands of literature to which the paper contributes,

[3]A null event is an event believed to have zero probability, and a nonnull event is an event believed to have nonzero probability.

including the tort law and economics literature, the behavioral law and economics literature, the unawareness literature, and the nascent literature on law and unawareness.]

The remainder of the paper is organized as follows. Section 2 presents the accident model—a unilateral accident model featuring multiple activities with fixed levels—and derives the equivalence result. Section 3 presents the unawareness model and provides relevant examples of new acts, new consequences, and new act-consequence links. Section 4 compares negligence and strict liability in a world with unawareness. It considers a simplified world with two acts, two consequences, quadratic care costs, and linear expected harm reduction, and separately analyzes the cases of a new act, a new consequence, and a new act-consequence link. Section 5 extends the analysis to a more general world with m acts, n consequences, convex care costs, and convex expected harm reduction. Section 6 discusses the results and suggests directions for future research. The Appendix collects the proofs of the propositions and corollaries stated but not proved in the body of the paper.

2 The Accident Model

There are two representative agents: an injurer and a victim. Both are risk neutral subjective expected utility maximizers. The agents are strangers and not parties to a contract or market transaction, and transaction costs are sufficiently high to preclude Coasian bargaining.

The injurer has available $m \geq 2$ activities, $f_1, \ldots, f_m$. Each activity has the potential to cause harm to the victim, though we assume the outcomes are independent across activities. That is, we assume the activities are independent experiments, akin to m one-armed bandits. We refer to this assumption below as *act independence*.[4]

There are $n \geq 2$ potential degrees of harm, $z_1, \ldots, z_n$, where $z_j \geq 0$ for all $j = 1, \ldots, n$. Activity f_i causes harm z_j with probability π_{ij}, where $\sum_{j=1}^{n} \pi_{ij} = 1$ for all $i = 1, \ldots, m$. Thus, activity f_i's expected harm is $\sum_{j=1}^{n} \pi_{ij} z_j$. In the absence of unawareness, the agents have correct beliefs about each harm probability π_{ij}.

[4] This is an important assumption which we revisit in Section 6.

The injurer engages in each available activity. For each activity f_i, the injurer, but not the victim, can take care to reduce the activity's expected harm to the victim. The injurer chooses a level of care $x_i \geq 0$ having cost $c(x_i)$. Being careless is costless, $c(0) = 0$, and the marginal cost of care is positive and increasing: $c'(x_i) > 0$ and $c''(x_i) > 0$ for all x_i. Taking care reduces the activity's expected harm at a nonincreasing rate: $h_i(x_i) \equiv \sum_{j=1}^{n} \pi_{ij} z_j \tau(x_i)$, where $\tau(x_i) \in (0, 1]$ for all x_i with $\tau(0) = 1$ and where $\tau'(x_i) < 0$ and $\tau''(x_i) \geq 0$ for all x_i.[5]

If activity f_i causes harm, the victim may be entitled to damages from the injurer, depending on the applicable tort liability rule. Under negligence, the victim is entitled to damages equal to the harm if the injurer's level of care was below the due care standard for the activity, $\overline{x}_i$, which is stipulated by the court. Under strict liability, the victim is entitled to damages equal to the harm irrespective of the injurer's level of care. We assume the injurer has the ability to pay any and all damages to which the victim may be entitled.

The social goal is to minimize the total social costs of the injurer's activities (the sum of the costs of care and the expected harms):

$$\underset{x_1,\ldots,x_m \geq 0}{\text{minimize}} \quad \sum_{i=1}^{m} c(x_i) + h_i(x_i).$$

The solution $\widetilde{x} = (\widetilde{x}_1, \ldots, \widetilde{x}_m)$ is given implicitly by the first order conditions

$$c'(\widetilde{x}_i) = -h_i'(\widetilde{x}_i), \quad i = 1, \ldots, m,$$

and is given explicitly by

$$\widetilde{x}_i = \xi^{-1}\left(\sum_{j=1}^{n} \pi_{ij} z_j\right), \quad i = 1, \ldots, m,$$

where ξ^{-1} denotes the inverse of $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$.[6] We refer to $\widetilde{x}_i$ as the efficient level

[5] We assume $c(\cdot)$ and $\tau(\cdot)$ are common knowledge but not activity specific. The later assumption is without loss of generality given the former assumption; we make the later assumption to simplify the notation.

[6] Note that $\xi'(x_i) = \frac{c'(x_i)t''(x_i) - t'(x_i)c''(x_i)}{[t'(x_i)]^2} > 0$ for all x_i; hence ξ is invertible.

of care for activity f_i. It is the level of care at which the marginal cost of care equals the marginal benefit (the marginal reduction in expected harm).

Under strict liability, the injurer's problem is identical to the social goal. This is because strict liability forces the injurer to internalize the total social costs of his activities. Hence, strict liability induces the injurer to take efficient care in each activity.

Under negligence, the injurer's problem is

$$\begin{array}{ll} \underset{x_1,\ldots,x_m \geq 0}{\text{minimize}} & \sum_{i=1}^m c(x_i) + h_i(x_i) 1(x_i < \overline{x}_i), \\ \text{where} & 1(x_i < \overline{x}_i) \equiv \begin{cases} 1 & \text{if } x_i < \overline{x}_i \\ 0 & \text{otherwise} \end{cases} \end{array}$$

and where $\overline{x}_i$ is the due care standard for activity f_i. If the court sets $\overline{x}_i = \widetilde{x}_i$ for all i, then the injurer takes efficient care in each activity. The reason is twofold. First, the injurer will not take more than the efficient level of care, because he faces no liability if his level of care equals or exceeds the efficient level. Second, the injurer will not take less than the efficient level of care, because then he faces strictly liability, which induces him to take efficient care.

The equivalence result follows immediately from the foregoing.

Theorem 1 (Equivalence Result) *The injurer will take efficient care in each activity under either negligence or strict liability, provided that, in the case of negligence, the court sets the due care standard for each activity equal to the efficient level of care for that activity.*

3 The Unawareness Model

We model unawareness and growing awareness à la Karni and Vierø (2013). The primitives of the model are a finite, nonempty set F of feasible *acts* and a finite, nonempty set Z of feasible *consequences*. In our setting, the feasible acts are the injurer's available activities and the feasible consequences are the potential harms to the victim.

States are functions from the set of acts to the set of consequences. A state assigns a consequence to each act. The set of all possible states, Z^F, defines the *conceivable state space*. With m acts and n consequences, there are n^m conceivable states.

The agents and the court (collectively, the *parties*) originally conceive the sets of acts and consequences to be $F = \{f_1, \ldots, f_m\}$ and $Z = \{z_1, \ldots, z_n\}$. The conceivable state space is $Z^F = \{s_1, \ldots, s_{n^m}\}$, where each state $s \in Z^F$ is a vector of length m, the ith element of which, s^i, is the consequence $z_j \in Z$ produced by act $f_i \in F$ in that state of the world.

An act-consequence link, or *link*, is a causal relationship between an act and a consequence. The conceivable state space admits all conceivable links. However, the parties may perceive one or more links as infeasible, which brings them to nullify the states that admit such link. We refer to these as null states. Taking only the nonnull states defines the *feasible state space*, $S \equiv Z^F \backslash N$, where $N \subset Z^F$ is the set of null states. When $N \neq \emptyset$, there are $\prod_{i=1}^m (n - \nu_i)$ feasible states, where ν_i denotes the number of nullified links involving act f_i.

The parties have common beliefs represented by a probability measure p on the conceivable state space, Z^F. The support set of p is the feasible state space, S. That is, the parties assign nonzero probability to each nonnull state and zero probability to each null state.

The parties may initially fail to conceive one or more acts or consequences or to perceive as feasible one or more conceivable links. We refer to such failures of conception or perception as *unawareness*. However, the parties may later discover a new act or consequence, which expands both the feasible state space and the conceivable state space, or a new link, which expands the feasible state space but not the conceivable state space.[7] We refer to such discoveries and expansions as *growing awareness*.

To illustrate, suppose $S = Z^F$ and the parties discover a new consequence, z_{n+1}. Then the set of potential harms becomes $\widehat{Z} = Z \cup \{z_{n+1}\}$ and the feasible and conceivable state spaces both expand to $\widehat{S} = \widehat{Z}^F = \{s_1, \ldots, s_{(n+1)^m}\}$, where each state remains a vector of length m. Alternatively, suppose the parties discover a new act, f_{m+1}. Then the set of

[7] To be clear, by "new" we mean "not previously conceived" in the case of acts and consquences, and "previously conceived but perceived as infeasible" in the case of links.

available activities becomes $\widehat{F} = F \cup \{f_{m+1}\}$ and the feasible and conceivable state spaces both expand to $\widehat{S} = Z^{\widehat{F}} = \{s_1, \ldots, s_{n^{(m+1)}}\}$, where each state now is a vector of length $m+1$. Lastly, suppose $S \subset Z^F$ because the parties initially perceive as infeasible the link from f_1 to z_n. Discovery of the link from f_1 to z_n does not alter the conceivable state space, but the feasible state space expands to coincide with the conceivable state space: $\widehat{S} = Z^F$.

In the wake of growing awareness, the parties' beliefs update in a way that preserves the relative likelihoods of the events in the original feasible state space (which are the nonnull events in the original conceivable state space). In each case of growing awareness, probability mass shifts proportionally away from the events in the original feasible state space to the new events in the expanded feasible state space. In the case of a new act or consequence, the new events in the expanded feasible state space are also new events in the expanded conceivable state space. In the case of a new link, the new events in the expanded feasible state space are the null events in the original conceivable state space that become nonnull.

Karni and Vierø refer to this updating as *reverse Bayesianism*. Let $\widehat{p}$ denote the parties' updated beliefs. Formally, reverse Bayesianism implies: (i) in the case of a new consequence or link, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$; and (ii) in the case of a new act, $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$ for all $s, t \in S$, where $E(s)$ denotes the event in $\widehat{S}$ that corresponds to state s in S; that is, $E(s) \equiv \{t \in \widehat{S} : t^i = s^i \text{ for all } i \neq m+1\}$ (assuming the new act is f_{m+1}).

The act independence assumption implies additional restrictions on $\widehat{p}$. Let $A_i(z_j) \subset \widehat{S}$ denote the event that f_i yields z_j; that is, $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$. We refer to events of this type as *act events*. Act independence implies $A_i(z_j) \perp A_{i'}(z_{j'})$ for all i and i' where $i \neq i'$ and all j and j'. Take any event $E \subseteq \widehat{S}$. We can express each state $s = (s^1, \ldots, s^m) \in E$ as the intersection of a unique collection of independent act events: $s = \bigcap_i A_i(s^i)$. It follows that $\widehat{p}(s) = \prod_i \widehat{p}\,(A_i(s^i))$ for all $s \in E$. Observe that growing awareness, whether it entails a new act, consequence, or link, gives rise to a new event $\Delta = \widehat{S} \backslash S$. Thus, in each case of growing awareness, act independence implies $\widehat{p}(s) = \prod_i \widehat{p}\,(A_i(s^i))$ for all $s \in \Delta$.

[TBA: Relevant examples of new acts, consequences, and links.]

4 Illustrative Results

In this section and the next, we compare and contrast negligence and strict liability in a world with unawareness. In both sections, we assume that when the parties are unaware of an act, consequence, or link, their beliefs, although incorrect with respect to the absolute likelihoods of events, are nevertheless correct with respect to the relative likelihoods of nonnull events. Without this assumption, the parties could not have correct beliefs when they become fully aware, which would be inconsistent with the standard accident model.

In this section, we consider a simplified world with two acts, $F = \{f_1, f_2\}$; two consequences, $Z = \{z_1, z_2\}$, where $z_1 = 0$ and $z_2 > 0$; quadratic care costs, $c(x_i) = (x_i)^2$; and linear expected harm reduction, $\tau(x_i) = (1 - x_i)$.[8] Our analysis of this simplified world illustrates all of the main ideas of the paper. In the next section, we show that the results extend to a more general world with m acts, n consequences, convex care costs, and convex expected harm reduction.

With two acts (activities), f_1 and f_2, and two consequences (harms), $z_1 = 0$ and $z_2 > 0$, the conceivable state space, Z^F, comprises four states: $s_1 = (0, 0)$, $s_2 = (0, z_2)$, $s_3 = (z_2, 0)$, and $s_4 = (z_2, z_2)$. Let $p_k \equiv p(s_k)$, $k = 1, \ldots, 4$, denote the parties' beliefs on Z^F. We can depict the original conceivable state space and the parties' beliefs as follows:

p	p_1	p_2	p_3	p_4
$F \backslash Z^F$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2 .

4.1 New Link

We start with the case of a new link. We assume the parties initially perceive activity f_1 as safe and activity f_2 as risky. That is, we assume they initially perceive the event $\{s_3, s_4\}$ as

[8] To preserve the condition $\tau(x_i) > 0$ for all x_i, we assume $x_i \in [0, 1)$ in this section.

infeasible (null). This implies $p_3 = p_4 = 0$. We can depict the original feasible state space, $S \subset Z^F$, as follows:

p	p_1	p_2
$F \backslash S$	s_1	s_2
f_1	0	0
f_2	0	z_2

Given S and p, the efficient levels of care are

$$\widetilde{x}_1 = 0 \quad \text{and} \quad \widetilde{x}_2 = \frac{p_2 z_2}{2}.$$

Under negligence, the court stipulates $\overline{x}_1 = \widetilde{x}_1$ and $\overline{x}_2 = \widetilde{x}_2$ as the due care standards for f_1 and f_2, respectively.

Suppose the parties discover that activity f_1 is risky. In particular, suppose that the injurer engages in f_1, that it results in harm z_2, and that the victim brings a tort suit against the injurer before the court. The feasible state space expands to coincide with the conceivable state space, $\widehat{S} = Z^F$, and the parties update their beliefs from p to $\widehat{p}$:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$
$F \backslash \widehat{S}$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

We assume that, by virtue of the suit, the parties learn that activity f_1 yields harm z_2 with probability $\delta > 0$.[9] Note that δ is the total probability of the new states in the expanded feasible state space. It is a measure of the likelihood of the event of which the parties were previously unaware. Thus, we interpret δ as the *degree of unawareness.*

[9] This is an important assumption which we defend in Section 6.

By reverse Bayesianism,

$$\frac{p_1}{p_2} = \frac{\widehat{p}_1}{\widehat{p}_2}.$$

In addition, $\delta = \widehat{p}_3 + \widehat{p}_4$ (by definition) and $\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_3 + \widehat{p}_4 = 1$ (by the unit measure axiom on $\widehat{S}$). Moreover, by act independence,

$$\widehat{p}_3 = (\widehat{p}_3 + \widehat{p}_4)(\widehat{p}_1 + \widehat{p}_3) \quad \text{and} \quad \widehat{p}_4 = (\widehat{p}_3 + \widehat{p}_4)(\widehat{p}_2 + \widehat{p}_4).$$

It follows that:

Proposition 1 $\quad \widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = \delta p_1, \quad$ *and* $\quad \widehat{p}_4 = \delta p_2.$

Note that p is the Bayesian update of $\widehat{p}$ conditional on the event $\{s_1, s_2\}$; hence the term *reverse Bayesianism*.

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are

$$\widehat{\widetilde{x}}_1 = \frac{(\widehat{p}_3 + \widehat{p}_4)\, z_2}{2} = \frac{\delta z_2}{2} \quad \text{and} \quad \widehat{\widetilde{x}}_2 = \frac{(\widehat{p}_2 + \widehat{p}_4) z_2}{2} = \frac{p_2 z_2}{2}.$$

Note that $\widehat{\widetilde{x}}_1 > \widetilde{x}_1$ but $\widehat{\widetilde{x}}_2 = \widetilde{x}_2$. Thus, the discovery that f_1 is risky necessitates the stipulation of a new due care standard for f_1 but not for f_2.

Under negligence, the court stipulates $\widehat{\overline{x}}_1 = \widehat{\widetilde{x}}_1$ as the new due care standard for f_1 and holds the injurer liable to pay damages of z_2 to the victim.[10] This makes the injurers and victims of the world aware that f_1 is risky. Moreover, the injurers and victims of the world can deduce δ from $\widehat{\overline{x}}_1$. Specifically, they can deduce that $\delta = 2\widehat{\widetilde{x}}_1 / z_2$. As a result, they can learn $\widehat{p}$ and $\widehat{h}_1(x_1) = \delta z_2 \tau(x_1)$, without expending additional resources to learn about δ. Knowledge of $\widehat{h}_1(x_1)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_2 to the victim. This makes the injurers and victims of the world aware that f_1 is risky. However,

[10] Recall that before the parties discover that f_1 is risky, $\overline{x}_1 = \widetilde{x}_1 = 0$. Under negligence, therefore, the injurer will have exercised no care in conjunction with f_1.

they cannot deduce δ or learn $\widehat{p}$ or $\widehat{h}_1(x_1)$. Without knowledge of $\widehat{h}_1(x_1)$, strict liability cannot induce the injurers of the world to take efficient care.

4.2 New Act

We next consider the case of a new act. We assume the original feasible state space coincides with the original conceivable state space (i.e., $S = Z^F$):

p	p_1	p_2	p_3	p_4
$F\backslash S$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2 .

Given S and p, the efficient levels of care are

$$\widetilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \widetilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates $\overline{x}_1 = \widetilde{x}_1$ and $\overline{x}_2 = \widetilde{x}_2$ as the due care standards for f_1 and f_2, respectively.

Suppose the parties discover a new activity, f_3, which they perceive as risky. In particular, suppose that the injurer discovers and engages in f_3, that it results in harm z_2, and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to eight states:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$	$\widehat{p}_5$	$\widehat{p}_6$	$\widehat{p}_7$	$\widehat{p}_8$
$F\backslash\widehat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
f_1	0	0	z_2	z_2	0	0	z_2	z_2
f_2	0	z_2	0	z_2	0	z_2	0	z_2
f_3	0	0	0	0	z_2	z_2	z_2	z_2 .

The expanded feasible state space contains two copies of the original feasible state space, one in which f_3 results in no harm and one in which f_3 results in harm z_2. Stated differently, the expanded space splits each of the original states into two depending on whether f_3 yields no harm or harm z_2. For each state in the original feasible state space there is a corresponding event in the expanded feasible state space. In particular, the event $\{s_1, s_5\} \in \widehat{S}$ corresponds to state $s_1 \in S$, the event $\{s_2, s_6\} \in \widehat{S}$ corresponds to state $s_2 \in S$, the event $\{s_3, s_7\} \in \widehat{S}$ corresponds to state $s_3 \in S$, and the event $\{s_4, s_8\} \in \widehat{S}$ corresponds to state $s_4 \in S$.[11]

We assume that, by virtue of the suit, the parties learn that activity f_3 yields harm z_2 with probability $\delta > 0$. By reverse Bayesianism,

$$\begin{aligned}
\frac{p_1}{p_2} &= \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_2 + \widehat{p}_6}, \qquad \frac{p_1}{p_3} = \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_3 + \widehat{p}_7}, \qquad \frac{p_1}{p_4} = \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_4 + \widehat{p}_8}, \\
\frac{p_2}{p_3} &= \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_3 + \widehat{p}_7}, \qquad \frac{p_2}{p_4} = \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_4 + \widehat{p}_8}, \quad \text{and} \quad \frac{p_3}{p_4} = \frac{\widehat{p}_3 + \widehat{p}_7}{\widehat{p}_4 + \widehat{p}_8}.
\end{aligned}$$

In addition, $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8$ (by definition) and $\widehat{p}_1 + \cdots + \widehat{p}_8 = 1$ (by the unit measure axiom on $\widehat{S}$). Moreover, by act independence,

$$\begin{aligned}
\widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\
\widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\
\widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8), \\
\text{and} \quad \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8).
\end{aligned}$$

It follows that:

Proposition 2 *$\widehat{p}_1 = (1-\delta)p_1$, $\widehat{p}_2 = (1-\delta)p_2$, $\widehat{p}_3 = (1-\delta)p_3$, $\widehat{p}_4 = (1-\delta)p_4$, $\widehat{p}_5 = \delta p_1$, $\widehat{p}_6 = \delta p_2$, $\widehat{p}_7 = \delta p_3$, and $\widehat{p}_8 = \delta p_4$.*

[11] Note that the conceivable state space also expands from four to eight states, so $\widehat{S} = Z^{\widehat{F}}$.

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are

$$
\begin{aligned}
\widehat{\widetilde{x}}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)\, z_2}{2} = \frac{(p_3 + p_4) z_2}{2}, \\
\widehat{\widetilde{x}}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)\, z_2}{2} = \frac{(p_2 + p_4) z_2}{2}, \\
\text{and} \quad \widehat{\widetilde{x}}_3 &= \frac{(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8)\, z_2}{2} = \frac{\delta z_2}{2}.
\end{aligned}
$$

Thus, the discovery of f_3 necessitates the stipulation of a new due care standard, $\widehat{\overline{x}}_3$, but it does necessitate the stipulation of new due care standards for f_1 or f_2.

Under negligence, the court stipulates $\widehat{\overline{x}}_3 = \widehat{\widetilde{x}}_3$ as the due care standard for the new activity f_3 and holds the injurer liable to pay damages of z_2 to the victim. This makes the injurers and victims of the world aware of f_3 (and that it is risky). Moreover, the injurers and victims of the world can deduce δ from $\widehat{\overline{x}}_3$; specifically, $\delta = 2\widehat{\widetilde{x}}_3 / z_2$. As a result, they can learn $\widehat{p}$ and $\widehat{h}_3(x_3) = \delta z_2 \tau(x_3)$, without expending additional resources to learn about δ. Knowledge of $\widehat{h}_1(x_1)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_2 to the victim. This makes the injurers and victims of the world aware of f_3 (and that it is risky). However, they cannot deduce δ or learn $\widehat{p}$ or $\widehat{h}_3(x_3)$. Without knowledge of $\widehat{h}_3(x_3)$, strict liability cannot induce the injurers of the world to take efficient care.

4.3 New Consequence

We last consider the case of a new consequence. As with the case of a new act, we assume $S = Z^F$:

p	p_1	p_2	p_3	p_4
$F \backslash S$	s_1	s_2	s_3	s_4
f_1	0	0	z_2	z_2
f_2	0	z_2	0	z_2

.

Given S and p, the efficient levels of care are

$$\widetilde{x}_1 = \frac{(p_3 + p_4)z_2}{2} \quad \text{and} \quad \widetilde{x}_2 = \frac{(p_2 + p_4)z_2}{2}.$$

Under negligence, the court stipulates $\overline{x}_1 = \widetilde{x}_1$ and $\overline{x}_2 = \widetilde{x}_2$ as the due care standards for f_1 and f_2, respectively.

Suppose the parties discover a new consequence, $z_3 > z_2$, which they link to f_1 and f_2. In particular, suppose that the injurer engages in f_1 and f_2, that each results in harm z_3, and that the victim brings a tort suit against the injurer before the court. The feasible state space expands from four to nine states:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$	$\widehat{p}_5$	$\widehat{p}_6$	$\widehat{p}_7$	$\widehat{p}_8$	$\widehat{p}_9$
$F\backslash\widehat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
f_1	0	0	z_2	z_2	z_3	z_3	0	z_2	z_3
f_2	0	z_2	0	z_2	0	z_2	z_3	z_3	z_3

The expanded feasible state space is characterized by three events, one in which f_1 results in no harm, one in which f_1 results in harm z_2, and one in which f_1 results in harm z_3. Each event contains three states, one in which f_2 results in no harm, one in which f_2 results in z_2, and one in which f_2 results in z_3.[12]

We assume that, by virtue of the suit, the parties learn that activity f_1 yields z_3 with probability $\alpha > 0$ and activity f_2 yields z_3 with probability $\beta > 0$. By reverse Bayesianism,

$$\begin{aligned} \frac{p_1}{p_2} &= \frac{\widehat{p}_1}{\widehat{p}_2}, & \frac{p_1}{p_3} &= \frac{\widehat{p}_1}{\widehat{p}_3}, & \frac{p_1}{p_4} &= \frac{\widehat{p}_1}{\widehat{p}_4}, \\ \frac{p_2}{p_3} &= \frac{\widehat{p}_2}{\widehat{p}_3}, & \frac{p_2}{p_4} &= \frac{\widehat{p}_2}{\widehat{p}_4}, \quad \text{and} & \frac{p_3}{p_4} &= \frac{\widehat{p}_3}{\widehat{p}_4}. \end{aligned}$$

In addition: by definition, $\alpha = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9$ and $\beta = \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$; by the unit measure axiom

[12] Note that the conceivable state space also expands from four to nine states, so $\widehat{S} = \widehat{Z}^F$.

on $\widehat{S}$, $\widehat{p}_1 + \cdots + \widehat{p}_6 = 1$; and by act independence,

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), \qquad \widehat{p}_6 = (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\ \widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), \qquad \widehat{p}_8 = (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9), \\ \text{and} \quad \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9). \end{aligned}$$

It follows that:

Proposition 3 *$\widehat{p}_1 = (1-\alpha)(1-\beta)p_1$, $\widehat{p}_2 = (1-\alpha)(1-\beta)p_2$, $\widehat{p}_3 = (1-\alpha)(1-\beta)p_3$, $\widehat{p}_4 = (1-\alpha)(1-\beta)p_4$, $\widehat{p}_5 = \alpha(1-\beta)(p_1 + p_3)$, $\widehat{p}_6 = \alpha(1-\beta)(p_2 + p_4)$, $\widehat{p}_7 = \beta(1-\alpha)(p_1 + p_2)$, $\widehat{p}_8 = \beta(1-\alpha)(p_3 + p_4)$, and $\widehat{p}_9 = \alpha\beta$.*

Note that the degree of unawareness is $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9 = \alpha + \beta - \alpha\beta$ and that $1 - \delta = (1-\alpha)(1-\beta)$. We can rewrite $\widehat{p}$ in terms of δ as follows:

Corollary 1 *$\widehat{p}_1 = (1-\delta)p_1$, $\widehat{p}_2 = (1-\delta)p_2$, $\widehat{p}_3 = (1-\delta)p_3$, $\widehat{p}_4 = (1-\delta)p_4$, $\widehat{p}_5 = \frac{\alpha}{1-\alpha}(1-\delta)(p_1 + p_3)$, $\widehat{p}_6 = \frac{\alpha}{1-\alpha}(1-\delta)(p_2 + p_4)$, $\widehat{p}_7 = \frac{\beta}{1-\beta}(1-\delta)(p_1 + p_2)$, $\widehat{p}_8 = \frac{\beta}{1-\beta}(1-\delta)(p_3 + p_4)$, and $\widehat{p}_9 = \alpha + \beta - \delta$.*

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are

$$\begin{aligned} \widehat{\widetilde{x}}_1 &= \frac{(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)z_2 + (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)z_3}{2} = \frac{(1-\alpha)(p_3 + p_4)z_2 + \alpha z_3}{2} \\ \text{and} \quad \widehat{\widetilde{x}}_2 &= \frac{(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_4)z_2 + (\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9)z_3}{2} = \frac{(1-\beta)(p_2 + p_4)z_2 + \beta z_3}{2}. \end{aligned}$$

Note that $\widehat{\widetilde{x}}_1 > \widetilde{x}_1$ and $\widehat{\widetilde{x}}_2 > \widetilde{x}_2$. Thus, the discovery of z_3 necessitates the stipulation of new due care standards for both f_1 and f_2.

Under negligence, the court stipulates $\widehat{\overline{x}}_1 = \widehat{\widetilde{x}}_1$ and $\widehat{\overline{x}}_2 = \widehat{\widetilde{x}}_2$ as the new due care standards for f_1 and f_2, respectively. The court holds the injurer liable to pay damages of z_3 to the victim with respect to each of f_1 and f_2. This makes the injurers and victims of the world aware of z_3 (and that it is linked to f_1 and f_2). Moreover, the injurers and victims of the world can deduce α and β (and, therefore, δ) from $\widehat{\widehat{x}}_1$ and $\widehat{\widehat{x}}_2$; specifically,

$$\alpha = \frac{p_3 z_2 - 2\widehat{\widehat{x}}_1 + p_4 z_2}{p_3 z_2 - z_3 + p_4 z_2} \quad \text{and} \quad \beta = \frac{p_2 z_2 - 2\widehat{\widehat{x}}_2 + p_4 z_2}{p_2 z_2 - z_3 + p_4 z_2}.$$

As a result, they can learn $\widehat{p}$ and

$$\begin{aligned} \widehat{h}_1(x_1) &= [(1-\alpha)(p_3 + p_4)z_2 + \alpha z_3]\tau(x_1) \\ \text{and} \quad \widehat{h}_2(x_2) &= [(1-\beta)(p_2 + p_4)z_2 + \beta z_3]\tau(x_2), \end{aligned}$$

without expending additional resources to learn about α and β. Knowledge of $\widehat{h}_1(x_1)$ and $\widehat{h}_2(x_2)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_3 for each instance of harm. This makes the injurers and victims of the world aware of z_3 (and that it is linked to f_1 and f_2). However, they cannot deduce α or β or learn $\widehat{p}$, $\widehat{h}_1(x_1)$, or $\widehat{h}_2(x_2)$. Without knowledge of $\widehat{h}_1(x_1)$ and $\widehat{h}_2(x_2)$, strict liability cannot induce the injurers of the world to take efficient care.

5 General Results

In this section, we show that our results extend to a more general world with m acts, n consequences, unspecified convex care costs, and unspecified convex expected harm reduction.

Let $F = \{f_1, \ldots, f_m\}$ be the set of activities and $Z = \{z_1, \ldots, z_n\}$ be the set of harms, where $0 \leq z_1 < z_2 < \cdots < z_n$. For each activity f_i, the cost of taking care $x_i \geq 0$ is $c(x_i)$, where $c(0) = 0$, $c'(x_i) > 0$, and $c''(x_i) > 0$ for all $x_i \geq 0$. Activity f_i's expected harm is $h_i(x_i) \equiv \sum_{j=1}^{n} \pi_{ij} z_j \tau(x_i)$, where (i) π_{ij} is the probability that f_i causes z_j and (ii) $\tau(x_i) \in (0, 1]$, $\tau(0) = 1$, $\tau'(x_i) < 0$, and $\tau''(x_i) \geq 0$ for all $x_i \geq 0$.[13]

[13] For example, we could have $t(x_i) = e^{-x_i}$.

Given F and Z, the conceivable state space is Z^F, where each state $s \in Z^F$ is a vector of length m, the ith element of which, s^i, is the harm $z_j \in Z$ caused by activity $f_i \in F$ in that state. The feasible state space is $S = Z^F \backslash N$, where $N \subset Z^F$ is the set of null states. Each state in N is induced by a nullified link between an activity f_i and a harm z_j.

Let p represent the parties' beliefs on Z^F. The support set of p is S. That is, $p(s) > 0$ for all $s \in S$ and $p(s) = 0$ for all $s \in N$.

Given S and p, the efficient levels of care are $\widetilde{x}_i = \xi^{-1}\left(\sum_{j=1}^{n} \pi_{ij} z_j\right)$, $i = 1, \ldots, m$, where (i) ξ^{-1} denotes the inverse of $\xi(x_i) \equiv -c'(x_i)/\tau'(x_i)$ and (ii) $\pi_{ij} = \sum\limits_{s \in S: s^i = z_j} p(s)$. Under negligence, the court stipulates $\overline{x}_i = \widetilde{x}_i$ as the due care standard for each activity f_i.

5.1 New Link

Assume $N \neq \emptyset$, so $S \subset Z^F$. Suppose the parties discover a new link from f_l to z_k for some $l \in \{1, \ldots, m\}$ and $k \in \{1, \ldots, n\}$. Let $\widehat{S}$ denote the expanded feasible state space and $\widehat{p}$ denote the parties' updated beliefs. In addition, let $\Delta = \widehat{S} \backslash S$. We assume that, by virtue of a tort litigation, the parties learn that f_l yields z_k with probability $\delta > 0$.

By reverse Bayesianism, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$. In addition, $\delta = \widehat{p}(\Delta)$ by definition and $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$ by the unit measure axiom on $\widehat{S}$. Moreover, by act independence, $\widehat{p}(s) = \prod_{i=1}^{m} \widehat{p}\left(A_i(s^i)\right)$ for all $s = (s^1, \ldots, s^m) \in \Delta$, where $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$ is the event that activity f_i yields harm z_j.

Given any $s \in \Delta$, let $L(s) \equiv \{t \in S : t^i = s^i \ \forall\ i \neq l\}$ denote the event in S that corresponds to $s \in \Delta$. It follows that:

Proposition 4 *In the case of a new link involving f_l, (i) $\widehat{p}(s) = (1-\delta)p(s)$ for all $s \in S$ and (ii) $\widehat{p}(s) = \delta p\left(L(s)\right)$ for all $s \in \Delta$.*

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are $\widehat{\widetilde{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n} \widehat{\pi}_{ij} z_j\right)$, $i = 1, \ldots, m$, where $\widehat{\pi}_{ij} = \sum\limits_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s)$. Specifically:

Proposition 5 *In the case of a new link from f_l to z_k, (i) $\widehat{\widetilde{x}}_l = \xi^{-1}\left(\sum_{j=1}^{n}(1-\delta)\pi_{lj}z_j + \delta z_k\right)$ and (ii) $\widehat{\widetilde{x}}_i = \widetilde{x}_i$ for all $i \neq l$.*

Corollary 2 *$\widehat{\widetilde{x}}_l = \widetilde{x}_l$ if and only if $z_k = \sum_{j=1}^{n}\pi_{lj}z_j$.*

Thus, the discovery that f_l can yield z_k necessitates the stipulation of a new due care standard for f_l (unless $z_k = \sum_{j=1}^{n}\pi_{lj}z_j$) but not for the other activities.

Under negligence, the court stipulates $\widehat{\overline{x}}_l = \widehat{\widetilde{x}}_l$ as the new due care standard for f_l (or restipulates $\widehat{\widetilde{x}}_l = \widetilde{x}_l$ if $z_k = \sum_{j=1}^{n}\pi_{lj}z_j$) and holds the injurer liable to pay damages of z_k to the victim if $\widehat{\overline{x}}_l > \overline{x}_l$. This, along with the victim's claim, makes the injurers and victims of the world aware that f_l can yield z_k. Moreover, the injurers and victims of the world can deduce δ from $\widehat{\overline{x}}_l$.

Proposition 6 *In the case of a new link from f_l to z_k, $\delta = \frac{c'(\widehat{\overline{x}}_l)+\sum_{j=1}^{n}\pi_{lj}z_j\tau'(\widehat{\overline{x}}_l)}{\sum_{j=1}^{n}\pi_{lj}z_j\tau'(\widehat{\overline{x}}_l)-z_k\tau'(\widehat{\overline{x}}_l)}$.*

As a result, they can learn $\widehat{p}$ and $\widehat{h}_l(x_l) = \sum_{j=1}^{n}\left[(1-\delta)\pi_{ij}z_j + \delta z_k\right]\tau(x_l)$, without expending additional resources to learn about δ. Knowledge of $\widehat{h}_l(x_l)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_k to the victim. This makes the injurers and victims of the world aware that f_l can yield z_k. However, they cannot deduce δ or learn $\widehat{p}$ or $\widehat{h}_l(x_l)$. Without knowledge of $\widehat{h}_l(x_l)$, strict liability cannot induce the injurers of the world to take efficient care.

5.2 New Act

Assume $S \subseteq Z^F$. Suppose the parties discover a new act, f_{m+1}. Let $\widehat{S}$ denote the expanded feasible state space and $\widehat{p}$ denote the parties' updated beliefs. We assume that, by virtue of a tort litigation, the parties learn that f_{m+1} yields z_j with probability $\delta_j > 0$ for all $j = 1, \ldots, n$.[14] Note that $\sum_{j=1}^{n}\delta_j = 1$.

[14] Assuming $\delta_j > 0$ for all $j = 1, \ldots, n$ is without loss of generality. We can deal with the case where $\delta_j = 0$ for some j by assuming $\delta_j > 0$ for the first $k < n$ and changing n to k as necessary in the statements below.

By reverse Bayesianism, $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$ for all $s, t \in S$, where $E(s) \equiv \{t \in \widehat{S} : t^i = s^i \ \forall \ i \neq m+1\}$ denotes the event in $\widehat{S}$ that corresponds to $s \in S$. Note that $\{E(s) : s \in S\}$ forms a partition of $\widehat{S}$ and that $|E(s)| = n$ for all $s \in S$. With a slight abuse of notation, index the states in each $E(s)$ by $j = 1, \ldots, n$.

By definition, $\delta_j = \widehat{p}(A_{m+1}(z_j))$, where $A_i(z_j) \equiv \{t \in \widehat{S} : t^i = z_j\}$ is the event that activity f_i yields harm z_j. In addition, $\sum_{s \in \widehat{S}} \widehat{p}(s) = 1$ by the unit measure axiom on $\widehat{S}$. Moreover, by act independence, $\widehat{p}(s) = \prod_{i=1}^{m+1} \widehat{p}\left(A_i(s^i)\right)$ for all $s = (s^1, \ldots, s^{m+1}) \in \widehat{S}$.

It follows that:

Proposition 7 *In the case of a new act f_{m+1}, for all $s \in S$ and corresponding $E(s) \subset \widehat{S}$, $\widehat{p}(s_j) = \delta_j p(s)$ for all $s_j \in E(s)$, $j = 1, \ldots, n$.*

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are $\widehat{\widetilde{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n} \widehat{\pi}_{ij} z_j\right)$, $i = 1, \ldots, m+1$, where $\widehat{\pi}_{ij} = \sum\limits_{s \in \widehat{S} : s^i = z_j} \widehat{p}(s)$. Specifically:

Proposition 8 *In the case of a new act f_{m+1}, (i) $\widehat{\widetilde{x}}_i = \widetilde{x}_i$ for all $i \neq m+1$ and (ii) $\widehat{\widetilde{x}}_{m+1} = \xi^{-1}\left(\sum_{j=1}^{n} \delta_j z_j\right)$.*

Thus, the discovery of f_{m+1} necessitates the stipulation of a new due care standard, $\widehat{\overline{x}}_{m+1}$, but it does necessitate the stipulation of new due care standards for $f_1, \ldots, f_m$.

Under negligence, the court stipulates $\widehat{\overline{x}}_{m+1} = \widehat{\widetilde{x}}_{m+1}$ as the due care standard for the new activity f_{m+1} and holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of f_{m+1} (and that it is risky). Although the injurers and victims of the world cannot separately deduce each δ_j from $\widehat{\overline{x}}_{m+1}$[15] they nevertheless can infer $\widehat{h}_{m+1}(x_{m+1})$ from $\widehat{\overline{x}}_{m+1}$, without expending additional resources to learn all δ_j.

Proposition 9 *In the case of a new act f_{m+1}, $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{\overline{x}}_{m+1})}{\tau'(\widehat{\overline{x}}_{m+1})}\tau(x_{m+1})$.*

[15] Note, however, that if each z_j is a different type of harm that requires a different type of care, then the court would stipulate a different due care standard $\widehat{\overline{x}}_{m+1,j}$ with respect to each z_j, in which case the injurers and victims of the world could separately deduce each δ_j.

Knowledge of $\widehat{h}_{m+1}(x_{m+1})$ is necessary to the world's injurers to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages to the victim. This makes the injurers and victims of the world aware of f_{m+1} (and that it is risky). However, they do not learn $\widehat{h}_{m+1}(x_{m+1})$. Without knowledge of $\widehat{h}_{m+1}(x_{m+1})$, strict liability cannot induce the injurers of the world to take efficient care.

5.3 New Consequence

Assume $S \subseteq Z^F$. Suppose the parties discover a new consequence, z_{n+1}. Let $\widehat{S}$ denote the expanded feasible state space and $\widehat{p}$ denote the parties' updated beliefs. In addition, let $\Delta = \widehat{S} \backslash S$ and $\delta = \widehat{p}(\Delta)$. We assume that, by virtue of a tort litigation, the parties learn that f_i yields z_{n+1} with probability $\alpha_i > 0$ for all $i = 1, \ldots, m$.[16] Note that $1 - \delta = \prod_{i=1}^{m}(1-\alpha_i)$.

By reverse Bayesianism, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s, t \in S$. In addition, $\alpha_i = \widehat{p}(A_i(z_{n+1}))$ by definition and $\sum_{s\in\widehat{S}} \widehat{p}(s) = 1$ by the unit measure axiom on $\widehat{S}$. Moreover, by act independence, $\widehat{p}(s) = \prod_{i=1}^{m} \widehat{p}\left(A_i(s^i)\right)$ for all $s = (s^1, \ldots, s^m) \in \Delta$.

Given any $s \in \Delta$, let $I(s) \equiv \{i \in \{1, \ldots, m\} : s^i = z_{n+1}\}$ denote the indices of the acts that yield z_{n+1} in that state of the world, let $\overline{I}(s) \equiv \{i \in \{1, \ldots, m\} : s^i \neq z_{n+1}\}$ denote the indices of the acts that do not yield z_{n+1} in that state of the world, and let $C(s) \equiv \{t \in S : t^i = s^i \; \forall \; i \in \overline{I}(s)\}$ denote the event in S that corresponds to $s \in \Delta$ on $\overline{I}(s)$.

It follows that:

Proposition 10 *In the case of a new consequence z_{n+1}, (i) $\widehat{p}(s) = \left(\prod_{i=1}^{m}(1-\alpha_i)\right) p(s)$ for all $s \in S$, (ii) $\widehat{p}(s) = \left(\prod_{i\in I(s)} \alpha_i\right) \left(\prod_{i\in\overline{I}(s)}(1-\alpha_i)\right) p\left(C(s)\right)$ for all $s \in \Delta$ such that $I(s) \subset \{1, \ldots, m\}$, and (iii) $\widehat{p}(s) = \prod_{i=1}^{m} \alpha_i$ for the $s \in \Delta$ such that $I(s) = \{1, \ldots, m\}$.*

Given $\widehat{S}$ and $\widehat{p}$, the efficient levels of care are $\widehat{\widehat{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j\right)$, $i = 1, \ldots, m$, where $\widehat{\pi}_{ij} = \sum\limits_{s\in\widehat{S}:s^i=z_j} \widehat{p}(s)$. Specifically:

[16] Assuming $\alpha_i > 0$ for all i is without loss of generality. We can deal with the case where $\alpha_i > 0$ for some i by assuming $\alpha_i > 0$ for the first $l < m$ and changing m to l as necessary in the statements below.

Proposition 11 *In the case of a new consequence* z_{n+1}, $\widehat{\widetilde{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n}(1-\alpha_i)\pi_{ij}z_j + \alpha_i z_{n+1}\right)$ *for all* $i = 1, \ldots, m$.

Corollary 3 $\widehat{\widetilde{x}}_i = \widetilde{x}_i$ *if and only if* $z_{n+1} = \sum_{j=1}^{n} \pi_{ij}z_j$.

Thus, the discovery of z_{n+1} necessitates the stipulation of new due care standards for each activity f_i such that $z_{n+1} \neq \sum_{j=1}^{n} \pi_{ij}z_j$.

Under negligence, the court stipulates $\widehat{\overline{x}}_i = \widehat{\widetilde{x}}_i$, $i = 1, \ldots, m$, as the new due care standards for $f_1, \ldots, f_m$ (or restipulates $\widehat{\overline{x}}_i = \widetilde{x}_i$ if $z_{n+1} = \sum_{j=1}^{n} \pi_{ij}z_j$) and holds the injurer liable to pay damages of z_{n+1} to the victim with respect to each activity f_i such that $\widehat{\overline{x}}_i > \overline{x}_i$. This, along with the victim's claims, makes the injurers and victims of the world aware of z_{n+1} (and that it is linked to $f_1, \ldots, f_m$). Moreover, the injurers and victims of the world can deduce $\alpha_1, \ldots, \alpha_m$ from $\widehat{\overline{x}}_1, \ldots, \widehat{\overline{x}}_m$.

Proposition 12 *In the case of a new consequence* z_{n+1}, $\alpha_i = \frac{c'(\widehat{\overline{x}}_l)+\sum_{j=1}^{n}\pi_{ij}z_j\tau'(\widehat{\overline{x}}_i)}{\sum_{j=1}^{n}\pi_{ij}z_j\tau'(\widehat{\overline{x}}_l)-z_{n+1}\tau'(\widehat{\overline{x}}_i)}$ *for all* $i = 1, \ldots, m$.

As a result, they can learn $\widehat{p}$ and $\widehat{h}_1(x_1), \ldots, \widehat{h}_m(x_m)$, without expending additional resources to learn about $\alpha_1, \ldots, \alpha_m$. Knowledge of $\widehat{h}_1(x_1), \ldots, \widehat{h}_m(x_m)$ is necessary to induce the injurers of the world to take efficient care.

Under strict liability, the court simply holds the injurer liable to pay damages of z_{n+1} to the victim with respect to each activity f_i. This makes the injurers and victims of the world aware of z_{n+1} (and that it is linked to $f_1, \ldots, f_m$). However, they can cannot deduce $\alpha_1, \ldots, \alpha_m$ or learn $\widehat{p}$ or $\widehat{h}_1(x_1), \ldots, \widehat{h}_m(x_m)$. Without knowledge of $\widehat{h}_1(x_1), \ldots, \widehat{h}_m(x_m)$, strict liability cannot induce the injurers of the world to take efficient care.

6 Discussion

[TBA: A discussion of the following points:

- *Main takeaway.* The new due care standard under negligence is like a public good. The social benefit of spreading awareness about δ is that the injurers of the world need not expend additional resources to develop this knowledge, which would be necessary to achieve optimal deterrence under strict liability. Negligence is akin to patents; both carry social costs (negligence is more costly to administer; patents create monopolies and deadweight loss) but provide social benefits in terms of knowledge spreading.

- *Reverse Bayesianism.* We assume reverse Bayesianism. Reasonable? Karni and Vierø (2013) provides axiomatic foundation. One can judge the theory by the axioms.

  - *Naive unawareness.* We assume the world is unaware that it is unaware. Reasonable? Karni and Vierø (2017) extends reverse Bayesianism to sophisticated unawareness.

- *Act independence.* Assuming act independence, in addition to reverse Bayesianism, allows us to pin down $\widehat{p}$. If we drop act independence: In the case of a new link or act, because the court learns δ, it still can stipulate the new due care standard and thereby spread awareness about δ. However, in the case of a new consequence, we would need to assume the parties learn the joint probability of new harm; otherwise the court could not stipulate the new due care standards. But even then negligence would spread only partial awareness, in the sense that the world would be able to deduce only lower and upper bounds on δ from the new due care standards; in other words, negligence would no longer resolve unawareness but rather reduce awareness to ambiguity.

- *Parties learn δ.* We can defend this assumption in (at least) two ways. First, because of the litigation, the parties have the incentive to expend resources to develop this knowledge. Second, the court may be able to infer δ from its docket of cases.

- *Court holds injurer liable and awards damages.* If the court does not do this (perhaps by recognizing a civil ex post facto doctrine which prohibits retroactive application of a due care standard in a negligence suit), this is not a problem in the case of a new act or link because the world already knows the set of potential harms. In the case of a new consequence, however, the stipulation of new due care standards is not sufficient to separately identify the new harm. But the victim's claims identify the new harm.

- *Cost and harm reduction functions are the same for all activities.* As long as the world knows them all, it is not a problem to relax this assumption and assume heterogeneity across activities.

- *Regulators and other actors who can spread awareness.* This is possible, but orthogonal. We are contributing to the negligence versus strict liability debate in tort law. Therefore, we consider a world where the tort system is the only mechanism for regulating risky activities and compare and contrast the two primary liability regimes.]

Appendix

Proof of Proposition 1

By reverse Bayesianism, the definition of δ, and the unit measure axiom on $\widehat{S}$, we have two linearly independent equations,

$$\widehat{p}_2 = \frac{p_2}{p_1}\widehat{p}_1 \quad \text{and} \quad \widehat{p}_1 + \widehat{p}_2 = 1 - \delta,$$

and two unknowns, $\widehat{p}_1$ and $\widehat{p}_2$. Substituting the first equation into the second, we have

$$\widehat{p}_1 + \frac{p_2}{p_1}\widehat{p}_1 = 1 - \delta,$$

which implies

$$\widehat{p}_1 = \frac{(1-\delta)p_1}{p_1 + p_2} = (1-\delta)p_1,$$

where the last equality follows from the unit measure axiom on S (which implies $p_1+p_2=1$). It follows that

$$\widehat{p}_2=\frac{p_2}{p_1}(1-\delta)p_1=(1-\delta)p_2.$$

By act independence assumption and the definition of δ, we have

$$\widehat{p}_3=\delta(\widehat{p}_1+\widehat{p}_3) \quad \text{and} \quad \widehat{p}_4=\delta(\widehat{p}_2+\widehat{p}_4),$$

which imply

$$\widehat{p}_3=\frac{\delta}{1-\delta}\widehat{p}_1 \quad \text{and} \quad \widehat{p}_4=\frac{\delta}{1-\delta}\widehat{p}_2.$$

It follows that

$$\widehat{p}_3=\frac{\delta}{1-\delta}(1-\delta)p_1=\delta p_1 \quad \text{and} \quad \widehat{p}_4=\frac{\delta}{1-\delta}(1-\delta)p_2=\delta p_2.$$

Proof of Proposition 2

Reverse Bayesianism implies three linearly independent conditions:[17]

$$\begin{aligned}
p_2(\widehat{p}_1+\widehat{p}_5) &= p_1(\widehat{p}_2+\widehat{p}_6),\\
p_3(\widehat{p}_1+\widehat{p}_5) &= p_1(\widehat{p}_3+\widehat{p}_7),\\
\text{and} \quad p_4(\widehat{p}_1+\widehat{p}_5) &= p_1(\widehat{p}_4+\widehat{p}_8).
\end{aligned}$$

Summing the left- and right-hand sides, and adding $p_1(\widehat{p}_1+\widehat{p}_5)$ to each side, yields

$$(p_1+p_2+p_3+p_4)(\widehat{p}_1+\widehat{p}_5)=(\widehat{p}_1+\cdots+\widehat{p}_8)p_1$$

[17]Note that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

By the unit measure axioms on S and $\widehat{S}$, we have $\widehat{p}_1 + \widehat{p}_5 = p_1$. Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 + \widehat{p}_5 = p_1, \quad \widehat{p}_2 + \widehat{p}_6 = p_2, \quad \widehat{p}_3 + \widehat{p}_7 = p_3, \quad \text{and} \quad \widehat{p}_4 + \widehat{p}_8 = p_4.$$

By act independence and the definition of δ, we have

$$\begin{aligned}
\widehat{p}_5 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_1 + \widehat{p}_5)\delta, \\
\widehat{p}_6 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_5 + \widehat{p}_6)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_2 + \widehat{p}_6)\delta, \\
\widehat{p}_7 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5 + \widehat{p}_7)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_3 + \widehat{p}_7)\delta, \\
\text{and} \quad \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_7 + \widehat{p}_8)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6 + \widehat{p}_8)(\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8) = (\widehat{p}_4 + \widehat{p}_8)\delta,
\end{aligned}$$

where the second equality follows from iterative application of act independence.[18] These imply

$$\widehat{p}_5 = \frac{\delta}{1-\delta}\widehat{p}_1, \quad \widehat{p}_6 = \frac{\delta}{1-\delta}\widehat{p}_2, \quad \widehat{p}_7 = \frac{\delta}{1-\delta}\widehat{p}_3, \quad \text{and} \quad \widehat{p}_8 = \frac{\delta}{1-\delta}\widehat{p}_4.$$

It follows that

$$\widehat{p}_1 + \frac{\delta}{1-\delta}\widehat{p}_1 = p_1, \quad \widehat{p}_2 + \frac{\delta}{1-\delta}\widehat{p}_2 = p_2, \quad \widehat{p}_3 + \frac{\delta}{1-\delta}\widehat{p}_3 = p_3, \quad \text{and} \quad \widehat{p}_4 + \frac{\delta}{1-\delta}\widehat{p}_4 = p_4.$$

These imply

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4,$$

which in turn imply

$$\widehat{p}_5 = \delta p_1, \quad \widehat{p}_6 = \delta p_2, \quad \widehat{p}_7 = \delta p_3, \quad \text{and} \quad \widehat{p}_8 = \delta p_4.$$

[18] For example, $(\widehat{p}_1+\widehat{p}_2+\widehat{p}_5+\widehat{p}_6)(\widehat{p}_1+\widehat{p}_3+\widehat{p}_5+\widehat{p}_7) = \widehat{p}(\{s_1, s_2, s_5, s_6\})\widehat{p}(\{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_2, s_5, s_6\} \cap \{s_1, s_3, s_5, s_7\}) = \widehat{p}(\{s_1, s_5\}) = \widehat{p}_1 + \widehat{p}_5$.

Proof of Proposition 3

Reverse Bayesianism implies three linearly independent conditions:[19]

$$\begin{aligned} p_2\widehat{p}_1 &= p_1\widehat{p}_2, \\ p_3\widehat{p}_1 &= p_1\widehat{p}_3, \\ \text{and} \quad p_4\widehat{p}_1 &= p_1\widehat{p}_4. \end{aligned}$$

Summing the left- and right-hand sides, and adding $p_1\widehat{p}_1$ to each side, yields

$$(p_1 + p_2 + p_3 + p_4)\widehat{p}_1 = (\widehat{p}_1 + \cdots + \widehat{p}_4)p_1.$$

By the unit measure axiom on S and $\delta \equiv \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$, we have $\widehat{p}_1 = (1-\delta)p_1$. Substituting this back into the reverse Bayesian conditions yields

$$\widehat{p}_1 = (1-\delta)p_1, \quad \widehat{p}_2 = (1-\delta)p_2, \quad \widehat{p}_3 = (1-\delta)p_3, \quad \text{and} \quad \widehat{p}_4 = (1-\delta)p_4.$$

By act independence and the definitions of α and β, we have

$$\begin{aligned} \widehat{p}_5 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5) = \alpha(\widehat{p}_1 + \widehat{p}_3 + \widehat{p}_5), \\ \widehat{p}_6 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6) = \alpha(\widehat{p}_2 + \widehat{p}_4 + \widehat{p}_6), \\ \widehat{p}_7 &= (\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_7), \\ \widehat{p}_8 &= (\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \beta(\widehat{p}_3 + \widehat{p}_4 + \widehat{p}_8), \\ \text{and} \quad \widehat{p}_9 &= (\widehat{p}_5 + \widehat{p}_6 + \widehat{p}_9)(\widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9) = \alpha\beta. \end{aligned}$$

[19] Note again that the six reverse Bayesianism conditions are not linearly independent. In particular, we can derive the last three conditions from the first three.

These imply

$$\begin{aligned}
\widehat{p}_5 &= \frac{\alpha}{1-\alpha}(\widehat{p}_1+\widehat{p}_3), \qquad && \widehat{p}_6 = \frac{\alpha}{1-\alpha}(\widehat{p}_2+\widehat{p}_4),\\
\widehat{p}_7 &= \frac{\beta}{1-\beta}(\widehat{p}_1+\widehat{p}_2), && \widehat{p}_8 = \frac{\beta}{1-\beta}(\widehat{p}_3+\widehat{p}_4),\\
\text{and} \quad \widehat{p}_9 &= \alpha\beta.
\end{aligned}$$

From $\widehat{p}_9 = \alpha\beta$, it follows that $\delta \equiv \widehat{p}_5+\widehat{p}_6+\widehat{p}_7+\widehat{p}_8+\widehat{p}_9 = \alpha+\beta-\alpha\beta$, which implies $1-\delta = (1-\alpha)(1-\beta)$. (Observe that $1-\delta = (1-\alpha)(1-\beta)$ also follows directly from act independence.) It follows that

$$\begin{aligned}
\widehat{p}_1 &= (1-\alpha)(1-\beta)p_1, \qquad && \widehat{p}_2 = (1-\alpha)(1-\beta)p_2,\\
\widehat{p}_3 &= (1-\alpha)(1-\beta)p_3, \quad \text{and} && \widehat{p}_4 = (1-\alpha)(1-\beta)p_4,
\end{aligned}$$

and in turn that

$$\begin{aligned}
\widehat{p}_5 &= \alpha(1-\beta)(p_1+p_3), \qquad && \widehat{p}_6 = \alpha(1-\beta)(\widehat{p}_2+\widehat{p}_4),\\
\widehat{p}_7 &= \beta(1-\alpha)(\widehat{p}_1+\widehat{p}_2), && \widehat{p}_8 = \beta(1-\alpha)(\widehat{p}_3+\widehat{p}_4),\\
\text{and} \quad \widehat{p}_9 &= \alpha\beta.
\end{aligned}$$

Proof of Corollary 1

We establish in the proof of Proposition 3 that

$$\begin{aligned}
\widehat{p}_1 &= (1-\delta)p_1, \qquad && \widehat{p}_2 = (1-\delta)p_2,\\
\widehat{p}_3 &= (1-\delta)p_3, \quad \text{and} && \widehat{p}_4 = (1-\delta)p_4.
\end{aligned}$$

We also observe that $1-\delta=(1-\alpha)(1-\beta)$ and $\delta=\alpha+\beta-\alpha\beta$. It follows that

$$\begin{aligned}
\widehat{p}_5 &= \alpha(1-\beta)(p_1+p_3)=\frac{\alpha}{1-\alpha}(1-\delta)(p_1+p_3),\\
\widehat{p}_6 &= \alpha(1-\beta)(\widehat{p}_2+\widehat{p}_4)=\frac{\alpha}{1-\alpha}(1-\delta)(\widehat{p}_2+\widehat{p}_4),\\
\widehat{p}_7 &= \beta(1-\alpha)(\widehat{p}_1+\widehat{p}_2)=\frac{\beta}{1-\beta}(1-\delta)(\widehat{p}_1+\widehat{p}_2),\\
\widehat{p}_8 &= \beta(1-\alpha)(\widehat{p}_3+\widehat{p}_4)=\frac{\beta}{1-\beta}(1-\delta)(\widehat{p}_3+\widehat{p}_4),\\
\text{and}\quad \widehat{p}_9 &= \alpha+\beta-\delta.
\end{aligned}$$

Proof of Proposition 4

(i) Take any $s\in S$. By reverse Bayesianism, we have $|S|-1$ linearly independent equations:

$$\widehat{p}(t)=\frac{p(t)}{p(s)}\widehat{p}(s),\quad \forall\, t\in S,\, t\neq s. \tag{4.1}$$

By the definition of δ and the unit measure axiom on $\widehat{S}$, we have

$$\textstyle\sum_{t\in S}\widehat{p}(t)=1-\delta. \tag{4.2}$$

Substituting (4.1) into (4.2), we have

$$\widehat{p}(s)+\sum_{t\in S:t\neq s}\frac{p(t)}{p(s)}\widehat{p}(s)=1-\delta,$$

which implies

$$\widehat{p}(s)=\frac{(1-\delta)p(s)}{\sum_{t\in S}p(t)}=(1-\delta)p(s), \tag{4.3}$$

where the last equality follows from the unit measure axiom on S.

(ii) Take any $s\in\Delta$. By act independence,

$$\widehat{p}(s)=\textstyle\prod_{i=1}^{m}\widehat{p}\left(A_i(s^i)\right).$$

Observe that $\widehat{p}\left(A_l(s^l)\right) = \widehat{p}\left(A_l(z_k)\right) = \delta$ and $\bigcap_{i\neq l} A_i(s^i) = L(s) \cup \{s\}$. It follows that

$$\begin{aligned}\widehat{p}(s) &= \delta \textstyle\prod_{i\neq l} \widehat{p}\left(A_i(s^i)\right) = \delta\widehat{p}\left(\bigcap_{i\neq l} A_i(s^i)\right) \\ &= \delta\widehat{p}\left(L(s)\cup\{s\}\right) = \delta\left[\widehat{p}\left(L(s)\right) + \widehat{p}(s)\right],\end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\delta}{1-\delta}\widehat{p}(L(s)). \tag{4.4}$$

Observe that $L(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \neq l$. It follows that

$$\widehat{p}(L(s)) = \textstyle\sum_{t\in L(s)} \widehat{p}(t) = \sum_{t\in L(s)}(1-\delta)p(t) = (1-\delta)p(L(s)), \tag{4.5}$$

where the second equality follows from (4.3). Substituting (4.5) back into (4.4), we have $\widehat{p}(s) = \delta p(L(s))$.

Proof of Proposition 5

(i) Observe that $\sum_{j=1}^{n} \widehat{\pi}_{lj} z_j = \sum_{j\neq k} \widehat{\pi}_{lj} z_j + \delta z_k$. By Proposition 4,

$$\sum_{j\neq k} \widehat{\pi}_{lj} z_j = \sum_{j\neq k}\left[\sum_{s\in\widehat{S}:s^l=z_j} \widehat{p}(s)\right] z_j = \sum_{j\neq k}\left[\sum_{s\in S:s^l=z_j}(1-\delta)p(s) + \sum_{s\in\Delta:s^l=z_j}\delta p\left(L(s)\right)\right] z_j.$$

Observe that $s^l = z_k$ for all $s \in \Delta$. It follows that, for all $j \neq k$,

$$\sum_{s\in\Delta:s^l=z_j}\delta p\left(L(s)\right) = 0.$$

Thus, we have

$$\sum_{j\neq k} \widehat{\pi}_{lj} z_j = \sum_{j\neq k}\left[\sum_{s\in S:s^l=z_j}(1-\delta)p(s)\right] z_j = \sum_{j\neq k}(1-\delta)\pi_{lj} z_j = \textstyle\sum_{j=1}^{n}(1-\delta)\pi_{lj} z_j,$$

where the last equality follows from $\pi_{lk} = 0$. Hence, $\sum_{j=1}^{n} \widehat{\pi}_{lj} z_j = \sum_{j=1}^{n}(1-\delta)\pi_{lj} z_j + \delta z_k$.

(ii) Take any $i \neq l$ and any j. By Proposition 4,

$$\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} (1-\delta)p(s) + \sum_{s \in \Delta: s^i = z_j} \delta p\left(L(s)\right).$$

Observe that $L(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \neq l$. Thus,

$$\sum_{s \in \Delta: s^i = z_j} p\left(L(s)\right) = \sum_{t \in S: t^i = z_j} p\left(t\right).$$

Hence,

$$\widehat{\pi}_{ij} = \sum_{s \in S: s^i = z_j} (1-\delta)p(s) + \sum_{s \in S: s^i = z_j} \delta p\left(s\right) = \sum_{s \in S: s^i = z_j} p(s) = \pi_{ij}.$$

It follows that $\widehat{\widetilde{x}}_i = \widetilde{x}_i$ for all $i \neq l$.

Proof of Corollary 2

By Proposition 5, $\xi(\widehat{\widetilde{x}}_l) = \sum_{j=1}^n (1-\delta)\pi_{lj} z_j + \delta z_k$. Observe that $\xi(\widetilde{x}_l) = \sum_{j=1}^n \pi_{lj} z_j$. It follows that $\xi(\widehat{\widetilde{x}}_l) = \xi(\widetilde{x}_l)$ if and only if $z_k = \sum_{j=1}^n \pi_{lj} z_j$. Because $\xi'(x_i) > 0$ for all x_i, we have $\widehat{\widetilde{x}}_l = \widetilde{x}_l$ if and only if $z_k = \sum_{j=1}^n \pi_{lj} z_j$.

Proof of Proposition 6

By Proposition 5 and $\widehat{\overline{x}}_l = \widehat{\widetilde{x}}_l$, we have $\xi(\widehat{\overline{x}}_l) = \sum_{j=1}^n (1-\delta)\pi_{lj} z_j + \delta z_k$. It follows that

$$\delta = \frac{\xi(\widehat{\overline{x}}_l) - \sum_{j=1}^n \pi_{lj} z_j}{z_k - \sum_{j=1}^n \pi_{lj} z_j}.$$

Observe that $\xi(\widehat{\overline{x}}_l) = -c'(\widehat{\overline{x}}_l)/\tau'(\widehat{\overline{x}}_l)$. Thus,

$$\delta = \frac{c'(\widehat{\overline{x}}_l) + \sum_{j=1}^n \pi_{lj} z_j \tau'(\widehat{\overline{x}}_l)}{\sum_{j=1}^n \pi_{lj} z_j \tau'(\widehat{\overline{x}}_l) - z_k \tau'(\widehat{\overline{x}}_l)}.$$

Proof of Proposition 7

Take any $s \in S$. By reverse Bayesianism, we have $|S| - 1$ linearly independent equations:

$$p(t)\widehat{p}(E(s)) = p(s)\widehat{p}(E(t)), \quad \forall\ t \in S,\, t \neq s.$$

Summing the left- and right-hand sides, and adding $p(s)\widehat{p}(E(s))$ to each side, yields

$$\widehat{p}(E(s)) \textstyle\sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(E(t).$$

By the unit measure axioms on S and $\widehat{S}$, we have

$$\widehat{p}(E(s)) = p(s). \tag{7.1}$$

Take any $s_j \in E(s)$, $j \in \{1, \ldots, n\}$. By act independence,

$$\widehat{p}(s_j) = \textstyle\prod_{i=1}^{m+1} \widehat{p}\left(A_i(s_j^i)\right).$$

Observe that $\widehat{p}\left(A_{m+1}(s_j^{m+1})\right) = \widehat{p}\left(A_{m+1}(z_j)\right) = \delta_j$ and $\bigcap_{i=1}^{m} A_i(s_j^i) = E(s)$. It follows that

$$\widehat{p}(s_j) = \delta_j \textstyle\prod_{i=1}^{m} \widehat{p}\left(A_i(s_j^i)\right) = \delta_j \widehat{p}\left(\bigcap_{i=1}^{m} A_i(s_j^i)\right) = \delta_j \widehat{p}\left(E(s)\right). \tag{7.2}$$

Substituting (7.1) into (7.2), we have $\widehat{p}(s_j) = \delta_j p(s)$.

Proof of Proposition 8

(i) Recall that $\{E(s) : s \in S\}$ forms a partition of $\widehat{S}$. Take any $i \neq m+1$ and any j. By Proposition 7,

$$\begin{aligned} \widehat{\pi}_{ij} &= \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} \left[\sum_{s_l \in E(s)} \widehat{p}(s_l) \right]. \\ &= \sum_{s \in S: s^i = z_j} \left[\sum_{l=1}^{n} \delta_l p(s) \right] = \sum_{s \in S: s^i = z_j} p(s) \left[\sum_{l=1}^{n} \delta_l \right]. \end{aligned}$$

Note that $\sum_{l=1}^{n} \delta_l = 1$. Thus, $\widehat{\pi}_{ij} = \sum\limits_{s \in S: s^i = z_j} p(s) = \pi_{ij}$. It follows that $\widehat{\widetilde{x}}_i = \widetilde{x}_i$ for all $i \neq m+1$.

(ii) By definition, $\widehat{\pi}_{m+1,j} = \delta_j$ for all $j = 1, \ldots, n$. Hence, $\widehat{\widetilde{x}}_{m+1} = \xi^{-1} \left(\sum_{j=1}^{n} \delta_j z_j \right)$.

Proof of Proposition 9

Observe that $\widehat{h}_{m+1}(x_{m+1}) = \sum_{j=1}^{n} \widehat{\pi}_{m+1,j} z_j \tau(x_{m+1})$ and $\widehat{\overline{x}}_{m+1} = \widehat{\widetilde{x}}_{m+1} = \xi^{-1} \left(\sum_{j=1}^{n} \widehat{\pi}_{m+1,j} z_j \right)$. The latter implies $\xi(\widehat{\overline{x}}_{m+1}) = \sum_{j=1}^{n} \widehat{\pi}_{m+1,j} z_j$. Thus, $\widehat{h}_{m+1}(x_{m+1}) = \xi(\widehat{\overline{x}}_{m+1}) \tau(x_{m+1})$. Recall that $\xi(x_i) \equiv -c(x_i)/\tau(x_i)$. Hence, $\widehat{h}_{m+1}(x_{m+1}) = -\frac{c'(\widehat{\overline{x}}_{m+1})}{\tau'(\widehat{\overline{x}}_{m+1})} \tau(x_{m+1})$.

Proof of Proposition 10

(i) Take any $s \in S$. By reverse Bayesianism, we have $|S| - 1$ linearly independent equations:

$$p(t)\widehat{p}(s) = p(s)\widehat{p}(t), \quad \forall\, t \in S,\, t \neq s.$$

Summing the left- and right-hand sides, and adding $p(s)\widehat{p}(s)$ to each side, yields

$$\widehat{p}(s) \sum_{t \in S} p(t) = p(s) \sum_{t \in S} \widehat{p}(t).$$

Observe that $\sum_{t \in S} p(t) = 1$ and $\sum_{t \in S} \widehat{p}(t) = 1 - \delta = \prod_{i=1}^{m}(1 - \alpha_i)$. Thus,

$$\widehat{p}(s) = (1 - \delta)p(s) = \left(\prod_{i=1}^{m}(1 - \alpha_i)\right) p(s). \tag{10.1}$$

(ii) Take any $s \in \Delta$ such that $I(s) = \{k\}$ for any $k \in \{1, \ldots, m\}$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^{m} \widehat{p}\left(A_i(s^i)\right).$$

Observe that $\widehat{p}\left(A_k(s^k)\right) = \widehat{p}\left(A_k(z_{n+1})\right) = \alpha_k$. Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \overline{I}(s)} \widehat{p}\left(A_i(s^i)\right).$$

Observe that $I(s) = \{k\}$ implies $\bigcap_{i \in \overline{I}(s)} A_i(s^i) = C(s) \cup \{s\}$. Hence,

$$\begin{aligned} \widehat{p}(s) &= \alpha_k \prod_{i \in \overline{I}(s)} \widehat{p}\left(A_i(s^i)\right) = \alpha_k \widehat{p}\left(\bigcap_{i \in \overline{I}(s)} A_i(s^i)\right) \\ &= \alpha_k \widehat{p}\left(C(s) \cup \{s\}\right) = \alpha_k \left(\widehat{p}\left(C(s)\right) + \widehat{p}\left(s\right)\right), \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}\left(C(s)\right). \tag{10.2}$$

Observe that $C(s)$ is the union of all $t \in S$ such that $t^i = s^i$ for all $i \in \overline{I}(s)$. It follows that

$$\widehat{p}(C(s)) = \sum_{t \in C(s)} \widehat{p}(t) = \sum_{t \in C(s)} (1 - \delta)p(t) = (1 - \delta)p(C(s)), \tag{10.3}$$

where the second equality follows from (10.1). Substituting (10.3) back into (10.2), we have

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k}(1 - \delta)p(C(s)) = \alpha_k \prod_{i \in \overline{I}(s)}(1 - \alpha_i)p(C(s)),$$

where the last equality follows from $1 - \delta = \prod_{i=1}^{m}(1 - \alpha_i)$.

Next take any $s \in \Delta$ such that $I(s) = \{k, l\}$ for any $\{k, l\} \subset \{1, \ldots, m\}$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^{m} \widehat{p}\left(A_i(s^i)\right).$$

Observe that $\widehat{p}\left(A_k(s^k)\right) = \widehat{p}\left(A_k(z_{n+1})\right) = \alpha_k$. Thus,

$$\widehat{p}(s) = \alpha_k \prod_{i \in \{\overline{I}(s) \cup \{l\}\}} \widehat{p}\left(A_i(s^i)\right).$$

Observe that $I(s) = \{k, l\}$ implies $\bigcap_{i \in \{\overline{I}(s) \cup \{l\}\}} A_i(s^i) = D(s) \cup \{s\}$, where $D(s) \equiv \{r \in \Delta : r^i = s^i \ \forall \ i \in \{\overline{I}(s) \cup \{l\}\}$. Hence,

$$\begin{aligned} \widehat{p}(s) &= \alpha_k \prod_{i \in \{\overline{I}(s) \cup \{l\}\}} \widehat{p}\left(A_i(s^i)\right) = \alpha_k \widehat{p}\left(\bigcap_{i \in \{\overline{I}(s) \cup \{l\}} A_i(s^i)\right) \\ &= \alpha_k \widehat{p}\left(D(s) \cup \{s\}\right) = \alpha_k \left(\widehat{p}\left(D(s)\right) + \widehat{p}\left(s\right)\right), \end{aligned}$$

which implies

$$\widehat{p}(s) = \frac{\alpha_k}{1 - \alpha_k} \widehat{p}\left(D(s)\right). \tag{10.4}$$

Observe further that $I(r) = \{l\}$ for all $r \in D(s)$. It follows that

$$\begin{aligned} \widehat{p}\left(D(s)\right) &= \sum_{t \in D(s)} \widehat{p}(t) \\ &= \sum_{t \in D(s)} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(t)) \\ &= \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)). \end{aligned} \tag{10.5}$$

Substituting (10.5) back into (10.4), we have

$$\begin{aligned} \widehat{p}(s) &= \frac{\alpha_k}{1 - \alpha_k} \frac{\alpha_l}{1 - \alpha_l} (1 - \delta) p(C(s)). \\ &= \alpha_k \alpha_l \prod_{i \in \overline{I}(s)} (1 - \alpha_i) p(C(s)). \end{aligned}$$

Proceeding in this fashion to consider $s \in \Delta$ such that $I(s)$ is an ι-element subset of

$\{1, \ldots, m\}$ for all $\iota = 3, \ldots, m-1$, we establish that

$$\widehat{p}(s) = \left(\prod_{i \in I(s)} \alpha_i\right) \left(\prod_{i \in \overline{I}(s)} (1 - \alpha_i)\right) p\left(C(s)\right)$$

for all $s \in \Delta$ such that $I(s) \subset \{1, \ldots, m\}$.

(iii) Take the $s \in \Delta$ such that $I(s) = \{1, \ldots, m\}$. By act independence,

$$\widehat{p}(s) = \prod_{i=1}^{m} \widehat{p}\left(A_i(s^i)\right).$$

Observe that $\widehat{p}\left(A_i(s^i)\right) = \widehat{p}\left(A_i(z_{n+1})\right) = \alpha_i$ for all $i \in I(s)$. Because $I(s) = \{1, \ldots, m\}$, we have $\widehat{p}(s) = \prod_{i=1}^{m} \alpha_i$.

Proof of Proposition 11

Take any $i \in \{1, \ldots, m\}$. Observe that

$$\widehat{\widehat{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n+1} \widehat{\pi}_{ij} z_j\right) = \xi^{-1}\left(\sum_{j=1}^{n} \widehat{\pi}_{ij} z_j + \alpha_i z_{n+1}\right). \tag{11.1}$$

Let $\Gamma(\alpha_l, s) \equiv \left(\prod_{l \in I(s)} \alpha_l\right) \left(\prod_{l \in \overline{I}(s)} (1 - \alpha_l)\right)$ for all $s \in \Delta$. By Proposition 10, for all $j \neq n+1$,

$$\widehat{\pi}_{ij} = \sum_{s \in \widehat{S}: s^i = z_j} \widehat{p}(s) = \sum_{s \in S: s^i = z_j} \prod_{l=1}^{m} (1 - \alpha_l) p(s) + \sum_{s \in \Delta: s^i = z_j} \Gamma(\alpha_l, s) p(C(s)).$$

Observe that

$$\sum_{s \in S: s^i = z_j} \prod_{l=1}^{m} (1 - \alpha_l) p(s) = \prod_{l=1}^{m} (1 - \alpha_l) \sum_{s \in S: s^i = z_j} p(s) = (1 - \delta)\pi_{ij}$$

and that

$$
\begin{aligned}
\sum_{s\in\Delta:s^i=z_j}\Gamma(\alpha_l,s)p(C(s)) &= \sum_{s\in\Delta:s^i=z_j}\left(\prod_{l\in I(s)}\alpha_l\right)\left(\prod_{l\in\bar{I}(s)}(1-\alpha_l)\right)p(C(s)) \\
&= \sum_{s\in\Delta:s^i=z_j}\frac{\prod_{l\in I(s)}\alpha_l}{\prod_{l\in I(s)}(1-\alpha_l)}(1-\delta)p(C(s)) \\
&= \sum_{I\subset\{\{1,\dots,m\}\setminus\{i\}\}}\frac{\prod_{l\in I}\alpha_l}{\prod_{l\in I}(1-\alpha_l)}(1-\delta)\pi_{ij} \\
&= \frac{1-\prod_{l\neq i}(1-\alpha_l)}{\prod_{l\neq i}(1-\alpha_l)}(1-\delta)\pi_{ij}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\widehat{\pi}_{ij} &= (1-\delta)\pi_{ij}+\frac{1-\prod_{l\neq i}(1-\alpha_l)}{\prod_{l\neq i}(1-\alpha_l)}(1-\delta)\pi_{ij} \\
&= (1-\delta)\pi_{ij}\left(1+\frac{1-\prod_{l\neq i}(1-\alpha_l)}{\prod_{l\neq i}(1-\alpha_l)}\right) \\
&= (1-\delta)\pi_{ij}\left(\frac{1}{\prod_{l\neq i}(1-\alpha_l)}\right)=(1-\delta)\pi_{ij}\left(\frac{1-\alpha_i}{1-\delta}\right)=(1-\alpha_i)\pi_{ij}. \qquad (11.2)
\end{aligned}
$$

Substituting (11.2) back into (11.1), we have $\widehat{\widetilde{x}}_i=\xi^{-1}\left(\sum_{j=1}^n(1-\alpha_i)\pi_{ij}z_j+\alpha_i z_{n+1}\right)$.

Proof of Corollary 3

By Proposition 11, $\xi(\widehat{\widetilde{x}}_i)=\sum_{j=1}^n(1-\alpha_i)\pi_{ij}z_j+(1-\alpha_i)z_{n+1}$. Observe that $\xi(\widetilde{x}_i)=\sum_{j=1}^n\pi_{ij}z_j$. It follows that $\xi(\widehat{\widetilde{x}}_i)=\xi(\widetilde{x}_i)$ if and only if $z_{n+1}=\sum_{j=1}^n\pi_{ij}z_j$. Because $\xi'(x_i)>0$ for all x_i, we have $\widehat{\widetilde{x}}_i=\widetilde{x}_i$ if and only if $z_{n+1}=\sum_{j=1}^n\pi_{ij}z_j$.

Proof of Proposition 12

Take any $i \in \{1, \ldots, m\}$. By Proposition 11, $\widehat{\overline{x}}_i = \widehat{\widetilde{x}}_i = \xi^{-1}\left(\sum_{j=1}^{n}(1-\alpha_i)\pi_{ij}z_j + \alpha_i z_{n+1}\right)$, which implies $\xi(\widehat{\overline{x}}_i) = \sum_{j=1}^{n}(1-\alpha_i)\pi_{ij}z_j + \alpha_i z_{n+1}$. It follows that

$$\alpha_i = \frac{\xi(\widehat{\overline{x}}_l) - \sum_{j=1}^{n}\pi_{ij}z_j}{z_{n+1} - \sum_{j=1}^{n}\pi_{ij}z_j}.$$

Observe that $\xi(\widehat{\overline{x}}_i) = -c'(\widehat{\overline{x}}_i)/\tau'(\widehat{\overline{x}}_i)$. Thus,

$$\alpha_i = \frac{c'(\widehat{\overline{x}}_l) + \sum_{j=1}^{n}\pi_{ij}z_j\tau'(\widehat{\overline{x}}_i)}{\sum_{j=1}^{n}\pi_{ij}z_j\tau'(\widehat{\overline{x}}_l) - z_{n+1}\tau'(\widehat{\overline{x}}_i)}.$$

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